

# A Method for Change Computation in Deductive Databases

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## Abstract

Change computation is an essential component in several capabilities of a deductive database, such as integrity constraints checking, materialized view maintenance and condition monitoring. In this paper, we present a general method for change computation, which is based on the use of transition and internal events rules. These rules explicitly define the insertions, deletions and modifications induced by a database update. Standard SLDNF resolution can be used to compute the induced changes, but other procedures could be used as well. Our method generalizes and extends previous work on change computation methods, and in some cases computes changes in a more efficient way.

## 1 Introduction

Deductive databases generalize relational databases by including not only base predicates (or relations), but also derived predicates (or views). A derived predicate is defined by means of one or more deductive rules.

In a deductive database, an update to base predicates may induce changes on one or more derived predicates. Change computation refers to the process of computing the changes induced by an update. The obvious way to compute changes would be to evaluate derived predicates in the states before and after the update, and to compute the differences between the two states. However, this can be very inefficient in most cases.

Efficient change computation is essential in several

capabilities of a deductive database, such as integrity constraints checking [BMM90], view maintenance [CeW91] and condition or situation monitoring [RCB+89], and several methods have been proposed in the past years. Some methods are specific for a particular problem, but others are more general. Methods for change computation can be analyzed in terms of: (1) What kind of changes are defined?; (2) When are changes computed?; and (3) How are changes computed?. Some methods make a distinction between "potential" changes and "real" changes induced by an update [Küc91], but we are only interested here in the computation of real changes, representing the net effect of an update.

Most of the methods have been developed as part of methods for integrity constraints checking. We can only mention two of them here, and refer to [BMM90] for a state-of-the-art survey. The method described in [BDM88,BrD88] defines insertion and deletion changes. An insertion occurs when a fact is true in the updated state and false before, while a deletion occurs when a fact is false in the updated state and true before. Thus, only "real" or "net" changes are computed, and the computation is performed before the database is updated. Changes are computed using expressions derived from an analysis of deductive rules. A similar method is given in [Oli91], where in some cases the derived expressions are more simplified.

Incremental methods for view maintenance also compute changes induced by an update on some materialized view. A method where views are specified using a standard query language, and considering arbitrary database updates, is given in [CeW91]. The method defines insertion and deletion changes of a materialized view as before, but changes are computed once base relations have been updated. Changes are computed by production rules [WiF90,WCL91] derived from an analysis of the view definition. Key constraints of base relations are also taken into account. As an example of a more specialized work, we mention [BCL89] where a method is presented to determine irrelevant updates (cannot change a view) and autonomously computable updates (the view can be updated using the view itself and the update).

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Change computation has also been used for situation monitoring in active databases. [BuC79] is one of the earlier works in this field, describing a method for detecting that a change in base relations cannot induce a change on an alerter. [RCB+89] presents a method developed as part of the HiPAC DBMS [CBB+89]. They consider not only insertion and deletions changes, as before, but also modification changes. Each tuple of a relation (base or derived) has an attribute that provides a unique immutable identifier, so that a tuple is modified if some of its attributes change. The method derives expressions for computing induced changes, again from an analysis of the definition of the derived predicate.

We present here a general method for change computation that can be applied in all database capabilities discussed above. The method takes into account key constraints of base and derived predicates. This allows us to define insertion, deletion and modification changes of derived predicates, where modification is defined as a change in some non-key argument of a predicate. The method computes the changes once the database has been updated. The changes are computed using expressions that are more simplified than those obtained in the previous methods, thus providing more efficient ways of change computation. The expressions are derived at compilation time, and evaluated when the database is updated.

The paper is organized as follows. Next Section defines basic concepts of deductive databases. In Section 3 we present the concept of internal event, a key concept of our method. Internal events capture in a natural way the notion of change. We also present the transition and internal events rules. Transition rules relate the old database state with the new state and the events that have occurred in a transition. Internal events rules define the conditions upon which an internal event happens. These rules are a particular application of the rules that we developed for the design of information systems [Oli89]. In Section 4 we show how internal events rules can be simplified. We give a set of simplifications that allow us to obtain simplified expressions for change computation. Then, in Section 5 we present our method for change computation, which can be based on the use of standard SLDNF resolution. We also point out some optimization techniques that can be applied. Our method is compared with some of the previous work in Section 6. Finally, we give in Section 7 the conclusions and point out future research.

## 2 Deductive Databases

A deductive database  $D$  consists of three finite sets: a set  $F$  of facts, a set  $R$  of deductive rules, and a set  $I$  of integrity constraints. A fact is a ground atom. The set of facts is called the Extensional Database (EDB), and the

set of deductive rules is called the Intensional Database (IDB).

We assume that database predicates are either base or derived. A base predicate appears only in the extensional database and (eventually) in the body of the deductive rules. A derived predicate appears only in the intensional database. Every database can be defined in this form [BaR 86].

We also assume that each database predicate (base or derived) has a non-null vector of arguments,  $k$ , that form a key for the predicate. We have then two types of predicates: those,  $P(k,x)$ , with key and non-key arguments and those,  $P(k)$ , with only key arguments, where both  $k$  and  $x$  are vectors.

### 2.1 Deductive Rules

A deductive rule is a formula of the form:

$$A \leftarrow L_1 \wedge \dots \wedge L_n \quad \text{with } n \geq 1$$

where  $A$  is an atom denoting the conclusion, and  $L_1, \dots, L_n$  are literals representing conditions. Each  $L_i$  is either an atom or a negated atom. Any variables in  $A, L_1, \dots, L_n$  are assumed to be universally quantified over the whole formula. We also assume that the terms in the conclusion must be distinct variables, and the terms in the conditions must be variables or constants.

Condition predicates may be ordinary or evaluable. The former are base or derived predicates, while the latter are predicates, such as the comparison or arithmetic predicates, that can be evaluated without accessing the database.

As usual, we require that the database before and after any update is *allowed* [Llo 87], that is any variable that occurs in a deductive rule has an occurrence in a positive condition of an ordinary predicate. This ensures that all negative conditions can be fully instantiated before they are evaluated by the "negation as failure" rule.

In this paper we deal with stratified databases [ABW 88]. A database is stratified if the set of its predicate symbols can be partitioned into a finite set of classes, say  $S_0, \dots, S_n$  such that for every deductive rule  $P \leftarrow$  Conditions, with  $P \in S_j$ ,

- (i) if  $Q \in S_i$  is the predicate symbol of a positive condition of  $P$ , then  $i \leq j$ , and
- (ii) if  $Q \in S_i$  is the predicate symbol of a negative condition of  $P$ , then  $i < j$ .

### 2.2 Integrity Constraints

An integrity constraint is a closed first-order formula that the database is required to satisfy. We deal with constraints that have the form of a denial:

$$\leftarrow L_1 \wedge \dots \wedge L_n \quad \text{with } n \geq 1$$

where the  $L_i$  are literals, and variables are assumed to be universally quantified over the whole formula. More general constraints can be transformed into this form as described in [LIT 84]. For the sake of uniformity, we (as in [DaW 89, Kow 78]) associate to each integrity constraint an inconsistency predicate  $Icn$  and thus it has the same form as the deductive rule. We call them integrity rules.

To enforce the concept of key we assume that associated to each  $P(k,x)$  there is a key integrity constraint that we define as:

$$Icn(k) \leftarrow P(k,x) \wedge P(k,x') \wedge x \neq x'$$

For example, if the EDB has the predicate  $Employee(emp,dept)$ , the key integrity rule stating that  $emp$  forms a key for the predicate would be:

$$Icl(\underline{emp}) \leftarrow Employee(\underline{emp},dept) \wedge Employee(\underline{emp},dept') \wedge dept \neq dept'$$

Note that, for clarity, we underline the key arguments of each predicate.

Keys of derived predicates can be deduced from the deductive rules of these predicates, using a procedure similar to that presented in [Dat90, chapters 19,20]

### 3 Transition and Internal Events Rules

In this section we define the events, a key concept in our method. We also explain how to derive the transition and internal events rules for a given database. These rules depend only on the deductive rules. They are independent from the base facts stored in the database. In a later section we will discuss the use of these rules for change computation.

We extend here the work reported in [Oli 91] in three directions. First, we define not only insertions and deletions, but also modifications of base and derived predicates. Usually, modifications are handled as deletions followed by insertions, but handling them as a base concept allows to improve efficiency. Second, we change the definition of transition rules to deal with the case where induced changes must be computed once base predicates have been updated. And third, we take into account key information.

#### 3.1 Events

Let  $D^\circ$  be a database,  $U$  an update and  $D$  the updated database. We say that  $U$  induces a transition from  $D^\circ$  (the old state) to  $D$  (the new state). We assume for the moment that  $U$  consists of an unspecified set of base facts that have been inserted, deleted and/or modified.

Due to the deductive rules,  $U$  may induce other updates on some derived predicates. Let  $P$  be a derived predicate, and let  $P^\circ$  and  $P$  denote the evaluation of  $P$  in  $D^\circ$  and  $D$ , respectively. Assuming that  $P^\circ(K,X)$  holds in  $D^\circ$ ,

where  $K$  and  $X$  are vectors of constants, three cases are possible:

- a.1  $P(K,X)$  also holds in  $D$
- a.2  $\neg\exists y$  such that  $P(K,y)$  holds in  $D$
- a.3  $\exists x'$ , such as  $X'$ , for which  $P(K,X')$  and  $X \neq X'$  holds in  $D$

and assuming that  $P(K,X)$  holds in  $D$ , three cases are also possible:

- b.1  $P^\circ(K,X)$  also holds in  $D^\circ$
- b.2  $\neg\exists y$  such that  $P^\circ(K,y)$  holds in  $D^\circ$
- b.3  $\exists x'$ , such as  $X'$ , for which  $P^\circ(K,X')$  and  $X \neq X'$  holds in  $D^\circ$

In case a.2 we say that a deletion internal event occurs in the transition, and we denote it by  $\delta P(K,X)$ . In case b.2 we say that an insertion internal events occurs in the transition, and we denote it by  $\iota P(K,X)$ . In cases a.3 and b.3 we say that a modification internal event occurs in the transition, and we denote it by  $\mu P(K,X,X')$  and  $\mu P(K,X',X)$ , respectively.

Formally, we associate to each derived predicate  $P$  an insertion and a deletion internal event predicate defined as:

- (1)  $\forall k,x (\iota P(k,x) \leftrightarrow P(k,x) \wedge \neg\exists y P^\circ(k,y))$
- (2)  $\forall k,x (\delta P(k,x) \leftrightarrow P^\circ(k,x) \wedge \neg\exists y P(k,y))$

where  $k$  and  $x$  are vectors of variables.

Furthermore, we associate to each derived predicate  $P$  with non-key arguments, a modification internal event predicate defined as:

- (3)  $\forall k,x,x' (\mu P(k,x,x') \leftrightarrow P^\circ(k,x) \wedge P(k,x') \wedge x \neq x')$

We handle the modification of a key as a deletion  $\delta P(k,x)$  and an insertion  $\iota P(k',x)$ .

From the above, we then have the equivalences:

- (4)  $\forall k,x (P^\circ(k,x) \leftrightarrow (P(k,x) \wedge \neg\iota P(k,x) \wedge \neg\mu P(k,x',x)) \vee \delta P(k,x) \vee \mu P(k,x,x'))$
- (5)  $\forall k,x (\neg P^\circ(k,x) \leftrightarrow (\neg P(k,x) \wedge \neg\delta P(k,x) \wedge \neg\mu P(k,x,x')) \vee \iota P(k,x) \vee \mu P(k,x',x))$

which relate the old state with the new state and the internal events induced in the transition.

We also use definition (1), (2) and (3) above for base predicates. In this case,  $\iota P$ ,  $\delta P$  and  $\mu P$  facts represent the external events (given by the update) corresponding to insertion, deletion and modifications of base facts, respectively. Therefore, we assume from now on that  $U$  consists of an unspecified set of insertion and/or deletion and/or modification external events. Notice that by (1), (2) and (3) we require:

- (6)  $\forall k, x (\iota P(k, x) \rightarrow \neg \exists y P^\circ(k, y))$  and  
 (7)  $\forall k, x (\delta P(k, x) \rightarrow P^\circ(k, x))$  and  
 (8)  $\forall k, x, x' (\mu P(k, x, x') \rightarrow P^\circ(k, x) \wedge x \neq x')$

also to hold for base predicates. Again, the  $\mu P$  predicate is defined only if  $P$  has non-key arguments. Due to this similar definition, we use sometimes the term "event" to denote either an internal or external event.

### Example 1

Consider the following database D:

#### Base Facts

Person(John,19), Person(Ann,15),  
 Person(Tom,20), Works(Tom)

#### Deductive rules

- (E.1)  $\text{Young}(p,a) \leftarrow \text{Person}(p,a) \wedge a < 20$   
 (E.2)  $\text{Student}(p,a) \leftarrow \text{Young}(p,a) \wedge \neg \text{Works}(p)$

Let the update be the set of external events  $U = \{ \iota \text{Person}(\text{Mary}, 15), \mu \text{Person}(\text{John}, 19, 20), \mu \text{Person}(\text{Ann}, 15, 16) \}$ . The internal events induced by  $U$  on Young are:  $\iota \text{Young}(\text{Mary}, 15)$ ,  $\delta \text{Young}(\text{John}, 19)$  and  $\mu \text{Young}(\text{Ann}, 15, 16)$  and the internal events induced on Student are:  $\iota \text{Student}(\text{Mary}, 15)$ ,  $\delta \text{Student}(\text{John}, 19)$  and  $\mu \text{Student}(\text{Ann}, 15, 16)$ .

## 3.2 Transition Rules

Let  $P$  be a derived predicate of the database. The definition of  $P$  consists of the rules in the database having  $P$  in the conclusion. Assume that there are  $m$  ( $m \geq 1$ ) such rules. For our purposes, we require to rename the predicate symbol in the conclusions of the  $m$  rules by  $P_1 \dots P_m$  and add the set of clauses:

$$P \leftarrow P_i \quad i = 1 \dots m$$

Consider now one of the rules  $P_i(k, x) \leftarrow L_1 \wedge \dots \wedge L_n$ . When the rule is to be evaluated in the old state its form is  $P_i^\circ(k, x) \leftarrow L_1^\circ \wedge \dots \wedge L_n^\circ$  where  $L_r^\circ$  ( $r = 1 \dots n$ ) is obtained by replacing the predicate  $Q$  of  $L_r$  by  $Q^\circ$ . Now, if we replace each literal in the body by its equivalent definition given in (4) or (5), we get a new rule, called a *transition rule*, which defines predicate  $P_i^\circ$  (old state) in terms of new state predicates and events.

More precisely, if  $L_r^\circ$  is an ordinary positive literal  $Q_r^\circ(k_r, x_r)$  we apply (4) and replace it by:

$$(Q_r(k_r, x_r) \wedge \neg \iota Q_r(k_r, x_r) \wedge \neg \mu Q_r(k_r, x_r, x_r')) \vee \delta Q_r(k_r, x_r) \vee \mu Q_r(k_r, x_r, x_r')$$

and if  $L_r^\circ$  is an ordinary negative literal  $\neg Q_r^\circ(k_r, x_r)$  we apply (5) and replace it by:

$$(\neg Q_r(k_r, x_r) \wedge \neg \delta Q_r(k_r, x_r) \wedge \neg \mu Q_r(k_r, x_r, x_r')) \vee \iota Q_r(k_r, x_r) \vee \mu Q_r(k_r, x_r, x_r')$$

If  $L_r^\circ$  is an evaluable predicate, we just replace  $L_r^\circ$  (positive or negative) by its new state version  $L_r$ .

It will be easier to refer to the resulting expression if we denote it by:

$$\begin{aligned} U(L_r^\circ) &= Q_r(k_r, x_r) \wedge \neg \iota Q_r(k_r, x_r) \wedge \neg \mu Q_r(k_r, x_r, x_r') \\ &\quad \text{if } L_r^\circ = Q_r^\circ(k_r, x_r) \\ &= \neg Q_r(k_r, x_r) \wedge \neg \delta Q_r(k_r, x_r) \wedge \neg \mu Q_r(k_r, x_r, x_r') \\ &\quad \text{if } L_r^\circ = \neg Q_r^\circ(k_r, x_r) \\ &= L_r \quad \text{if } L_r^\circ \text{ is evaluable} \end{aligned}$$

$$\begin{aligned} D(L_r^\circ) &= \delta Q_r(k_r, x_r) \quad \text{if } L_r^\circ = Q_r^\circ(k_r, x_r) \\ &= \iota Q_r(k_r, x_r) \quad \text{if } L_r^\circ = \neg Q_r^\circ(k_r, x_r) \end{aligned}$$

$$\begin{aligned} M(L_r^\circ) &= \mu Q_r(k_r, x_r, x_r') \quad \text{if } L_r^\circ = Q_r^\circ(k_r, x_r) \\ &= \mu Q_r(k_r, x_r, x_r') \quad \text{if } L_r^\circ = \neg Q_r^\circ(k_r, x_r) \end{aligned}$$

Notice that all variables  $x_r'$  are new, that is, not used before.

$U(L_r^\circ)$ ,  $D(L_r^\circ)$  and  $M(L_r^\circ)$  express condition for which  $L_r^\circ$  is true.  $U(L_r^\circ)$  corresponds to the case in which  $L_r^\circ$  is Unchanged in the transition.  $D(L_r^\circ)$  corresponds to the case in which  $L_r^\circ$  is Deleted, while  $M(L_r^\circ)$  corresponds to the case when  $L_r^\circ$  is Modified.

With this notation we then have:

$$(9) P_i^\circ(k, x) \leftrightarrow \bigwedge_{r=1}^{r=n} [U(L_r^\circ) \vee D(L_r^\circ) \vee M(L_r^\circ) | U(L_r^\circ)]$$

where the first option is taken if  $L_r^\circ$  is an ordinary literal and the second one if  $L_r^\circ$  is evaluable. After distributing  $\wedge$  over  $\vee$ , we get an equivalent set of transition rules, each of them with the general form:

$$(10) P_i^\circ(k, x) \leftarrow \bigwedge_{r=1}^{r=n} [U(L_r^\circ) | D(L_r^\circ) | M(L_r^\circ)] \quad j = 1 \dots \alpha$$

$$(11) P_i^\circ(k, x) \leftarrow P_i^\circ(k, x) \quad j = 1 \dots \alpha$$

with  $\alpha = 3^{n k_i} * 2^{k_i}$ , where  $n k_i$  is the number of ordinary literals with non-key arguments, and  $k_i$  is the number of ordinary literals with only key arguments.

In the above set of rules (10) it will be useful to assume that the rule corresponding to  $j = 1$  is:

$$(12) P_i^\circ(k, x) \leftarrow U(L_{1,1}^\circ) \wedge \dots \wedge U(L_n^\circ)$$

### Example 2

The transition rules corresponding to derived predicates Young and Student defined in Example 1 are:

$$(E.3) \text{Young}_{1,1}^\circ(p,a) \leftarrow \text{Person}(p,a) \wedge \neg \iota \text{Person}(p,a) \wedge \neg \mu \text{Person}(p,a,a) \wedge a < 20$$

$$(E.4) \text{Young}_{1,2}^\circ(p,a) \leftarrow \delta \text{Person}(p,a) \wedge a < 20$$

$$(E.5) \text{Young}_{1,3}^\circ(p,a) \leftarrow \mu \text{Person}(p,a,a) \wedge a < 20$$

with:

$$(E.6 \dots 8) \text{Young}_j^\circ(p,a) \leftarrow \text{Young}_{1,j}^\circ(p,a) \quad j = 1 \dots 3$$

$$(E.9) \text{Young}(p,a) \leftarrow \text{Young}_1^\circ(p,a)$$

- (E.10)  $\text{Student}^{\circ}_{1,1}(\underline{p},a) \leftarrow \text{Young}(\underline{p},a) \wedge \neg \iota \text{Young}(\underline{p},a) \wedge \neg \mu \text{Young}(\underline{p},a',a) \wedge \neg \text{Works}(\underline{p}) \wedge \neg \delta \text{Works}(\underline{p})$
- (E.11)  $\text{Student}^{\circ}_{1,2}(\underline{p},a) \leftarrow \text{Young}(\underline{p},a) \wedge \neg \iota \text{Young}(\underline{p},a) \wedge \neg \mu \text{Young}(\underline{p},a',a) \wedge \iota \text{Works}(\underline{p})$
- (E.12)  $\text{Student}^{\circ}_{1,3}(\underline{p},a) \leftarrow \delta \text{Young}(\underline{p},a) \wedge \neg \text{Works}(\underline{p}) \wedge \neg \delta \text{Works}(\underline{p})$
- (E.13)  $\text{Student}^{\circ}_{1,4}(\underline{p},a) \leftarrow \delta \text{Young}(\underline{p},a) \wedge \iota \text{Works}(\underline{p})$
- (E.14)  $\text{Student}^{\circ}_{1,5}(\underline{p},a) \leftarrow \mu \text{Young}(\underline{p},a',a) \wedge \neg \text{Works}(\underline{p}) \wedge \neg \delta \text{Works}(\underline{p})$
- (E.15)  $\text{Student}^{\circ}_{1,6}(\underline{p},a) \leftarrow \mu \text{Young}(\underline{p},a',a) \wedge \iota \text{Works}(\underline{p})$

with:

- (E.16...21)  $\text{Student}^{\circ}_j(\underline{p},a) \leftarrow \text{Student}^{\circ}_{1,j}(\underline{p},a) \quad j = 1 \dots 6$
- (E.22)  $\text{Student}^{\circ}(\underline{p},a) \leftarrow \text{Student}^{\circ}_1(\underline{p},a)$

Some transition and internal events rules are not allowed, due to the presence of some negative literals in their bodies. This could be solved with a minor transformation. For example, in rule E.3 replace  $\neg \mu \text{Person}(\underline{p},a',a)$  by  $\neg \text{Aux}(\underline{p},a)$  and add the rule:  $\text{Aux}(\underline{p},a) \leftarrow \mu \text{Person}(\underline{p},a',a)$ .

### 3.3 Insertion Internal Events Rules

Let  $P$  be a derived predicate. Insertion internal events of  $P$  were defined in (1) as:

$$\forall \mathbf{k}, \mathbf{x} \quad (\iota P(\mathbf{k}, \mathbf{x}) \leftrightarrow P(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P^{\circ}(\mathbf{k}, y))$$

If there are  $m$  rules for predicate  $P$ , we have:

- (13)  $\iota P(\mathbf{k}, \mathbf{x}) \leftarrow P_i(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P^{\circ}_1(\mathbf{k}, y) \wedge \dots \wedge \neg \exists y P^{\circ}_m(\mathbf{k}, y) \quad i=1 \dots m$

Notice that  $P_i(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P^{\circ}_i(\mathbf{k}, y)$  represent insertion events of predicate  $P_i$ . Thus, we have:

- (14)  $\iota P(\mathbf{k}, \mathbf{x}) \leftarrow \iota P_i(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P^{\circ}_1(\mathbf{k}, y) \wedge \dots \wedge \neg \exists y P^{\circ}_{i-1}(\mathbf{k}, y) \wedge \neg \exists y P^{\circ}_{i+1}(\mathbf{k}, y) \wedge \dots \wedge \neg \exists y P^{\circ}_m(\mathbf{k}, y) \quad i=1 \dots m$

- (15)  $\iota P_i(\mathbf{k}, \mathbf{x}) \leftarrow P_i(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P^{\circ}_i(\mathbf{k}, y) \quad i=1 \dots m$

Rules (14) and (15) are called *insertion internal events rules* of predicate  $P$  and  $P_i$ , respectively. They allow us to deduce which  $\iota P$  and  $\iota P_i$  facts (induced insertions) happen in a transition.

In section 4.3 we show how rules (15) can be simplified.

### 3.4 Deletion Internal Events Rules

Let  $P$  be a derived predicate. Deletion internal events for  $P$  were defined in (2) as:

$$\forall \mathbf{k}, \mathbf{x} \quad (\delta P(\mathbf{k}, \mathbf{x}) \leftrightarrow P^{\circ}(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P(\mathbf{k}, y))$$

If there are  $m$  rules for predicate  $P$ , we then have:

- (16)  $\delta P(\mathbf{k}, \mathbf{x}) \leftarrow P^{\circ}_i(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P_1(\mathbf{k}, y) \wedge \dots \wedge \neg \exists y P_i(\mathbf{k}, y) \wedge \dots \wedge \neg \exists y P_m(\mathbf{k}, y) \quad i=1 \dots m$

Note that  $P^{\circ}_i(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P_i(\mathbf{k}, y)$  represent deletion events of predicate  $P_i$ . Therefore, we have:

- (17)  $\delta P(\mathbf{k}, \mathbf{x}) \leftarrow \delta P_i(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P_1(\mathbf{k}, y) \wedge \dots \wedge \neg \exists y P_{i-1}(\mathbf{k}, y) \wedge \neg \exists y P_{i+1}(\mathbf{k}, y) \wedge \dots \wedge \neg \exists y P_m(\mathbf{k}, y) \quad i=1 \dots m$

- (18)  $\delta P_i(\mathbf{k}, \mathbf{x}) \leftarrow P^{\circ}_i(\mathbf{k}, \mathbf{x}) \wedge \neg \exists y P_i(\mathbf{k}, y) \quad i=1 \dots m$

Rules (17) and (18) are called *deletion internal events rules* of predicate  $P$  and  $P_i$ , respectively. They allow us to deduce which  $\delta P$  and  $\delta P_i$  facts (induced deletions) happen in a transition.

In section 4.1 we show how rules (18) can be simplified.

### 3.5 Modification Internal Events Rules

Let  $P$  be a derived predicate. Modification internal events for  $P$  were defined in (3) as:

$$\forall \mathbf{k}, \mathbf{x}, \mathbf{x}' \quad (\mu P(\mathbf{k}, \mathbf{x}, \mathbf{x}') \leftrightarrow P^{\circ}(\mathbf{k}, \mathbf{x}) \wedge P(\mathbf{k}, \mathbf{x}') \wedge \mathbf{x} \neq \mathbf{x}')$$

If there are  $m$  rules for predicate  $P$  we then have:

- (20)  $\mu P(\mathbf{k}, \mathbf{x}, \mathbf{x}') \leftarrow P^{\circ}_i(\mathbf{k}, \mathbf{x}) \wedge P_h(\mathbf{k}, \mathbf{x}') \wedge \mathbf{x} \neq \mathbf{x}' \quad i, h=1 \dots m$

Notice that  $P^{\circ}_i(\mathbf{k}, \mathbf{x}) \wedge P_i(\mathbf{k}, \mathbf{x}') \wedge \mathbf{x} \neq \mathbf{x}'$  represent modifications events of predicate  $P_i$ . Therefore, we have:

- (21)  $\mu P(\mathbf{k}, \mathbf{x}, \mathbf{x}') \leftarrow P^{\circ}_i(\mathbf{k}, \mathbf{x}) \wedge P_h(\mathbf{k}, \mathbf{x}') \wedge \mathbf{x} \neq \mathbf{x}' \quad i=1 \dots m, h=1 \dots m \text{ except } i$

- (22)  $\mu P(\mathbf{k}, \mathbf{x}, \mathbf{x}') \leftarrow \mu P_i(\mathbf{k}, \mathbf{x}, \mathbf{x}') \quad i=1 \dots m$

- (23)  $\mu P_i(\mathbf{k}, \mathbf{x}, \mathbf{x}') \leftarrow P^{\circ}_i(\mathbf{k}, \mathbf{x}) \wedge P_i(\mathbf{k}, \mathbf{x}') \wedge \mathbf{x} \neq \mathbf{x}' \quad i=1 \dots m$

Rules (21)-(22) and (23) are called *modification internal events rules* of predicate  $P$  and  $P_i$ , respectively. They allow us to deduce which  $\mu P$  and  $\mu P_i$  facts (induced modifications) happen in a transition. Notice that rules (21) are defined only when  $m > 1$ .

We show in section 4.2 how rules (21) and (23) can be simplified.

#### Example 3

The insertion, deletion and modification internal events rules corresponding to predicate  $\text{Young}$  defined in Example 1 are (the rules for  $\text{Student}$  are similar):

- (E.23)  $\iota \text{Young}(\underline{p},a) \leftarrow \text{Young}(\underline{p},a) \wedge \neg \exists y \text{Young}^{\circ}(\underline{p},y)$
- (E.24)  $\delta \text{Young}(\underline{p},a) \leftarrow \text{Young}^{\circ}(\underline{p},a) \wedge \neg \exists y \text{Young}(\underline{p},y)$
- (E.25)  $\mu \text{Young}(\underline{p},a,a') \leftarrow \text{Young}^{\circ}(\underline{p},a) \wedge \text{Young}(\underline{p},a') \wedge a \neq a'$

with the transition rules for  $\text{Young}^{\circ}(\underline{p},a)$  given in Example 2.

## 4 Simplification of Internal Events Rules

In this section we introduce the simplifications that can be applied to the deletion, insertion and modification internal events rules. As mentioned in section 3, we can often simplify and even remove some of these rules. Applying these simplifications, we obtain a set of rules semantically equivalent to the former but with a smaller evaluation cost. In fact, we will see in section 6 that the application of our simplifications produces expressions that are more optimized than those obtained by other methods.

We need to introduce first some terminology. If  $P$  is a predicate defined with a single rule, we denote by  $B(P)$  the body of its defining rule.  $B(P)$  is a conjunction of one or more literals. We also denote by  $A \setminus B$   $A$  without  $B$  (true if  $A=B$ ).

### 4.1 Simplification of Deletion Internal Events Rules

Deletion internal events rules for predicate  $P_i$  were defined in (18) as:

$$\delta P_i(k, x) \leftarrow P_i^\circ(k, x) \wedge \neg \exists y P_i(k, y) \quad i = 1 \dots m$$

replacing  $P_i^\circ(k, x)$  by its equivalent definition given in (11) we get:

$$(24) \quad \delta P_i(k, x) \leftarrow P_i^\circ(k, x) \wedge \neg \exists y P_i(k, y) \\ i = 1 \dots m, j = 1 \dots \alpha$$

We can remove from (24) the rules corresponding to  $j = 1$ , which have the form given in (12), since  $P_i^\circ(k, x) \rightarrow P_i(k, x)$ . We can then reduce the set (24) to:

$$(25) \quad \delta P_i(k, x) \leftarrow P_i^\circ(k, x) \wedge \neg \exists y P_i(k, y) \\ i = 1 \dots m, j = 2 \dots \alpha$$

which can be rewritten as:

$$(26) \quad \delta P_i(k, x) \leftarrow B(P_i^\circ) \wedge \neg \exists y B(P_i) \sigma \\ i = 1 \dots m, j = 2 \dots \alpha$$

where  $\sigma$  is the substitution  $\{y/x, z'/z\}$ ,  $z$  is the set of variables in  $B(P_i)$  except  $k$  and  $x$ ; and  $z'$  are new variables.

There are several simplifications that can be applied to rules (26) above. All of them are based on the analysis of the relationship between a literal  $L_h(k_h, u_h)$  in  $B(P_i^\circ)$  and the corresponding literal in  $B(P_i) \sigma$ . The result of this simplification can be either a reduced form of the expression  $\neg \exists y B(P_i) \sigma$  or a removal of the rule.

#### Deletion of a positive literal

If  $\delta Q_h(k_h, u_h)$  is a literal in  $B(P_i^\circ)$  and  $Q_h(k_h, v_h)$  is the corresponding literal in  $B(P_i) \sigma$ , then the expression  $\neg \exists y B(P_i) \sigma$  can be removed from (26). Notice that  $u_h, v_h$

can be null.

**Proof:** By (2),  $\delta Q_h(k_h, u_h) \rightarrow \neg \exists y Q_h(k_h, y)$  and thus  $Q_h(k_h, v_h)$  is false.  $\oplus$

#### Insertion of a negative literal

If  $\iota Q_h(k_h)$  is a literal in  $B(P_i^\circ)$  and  $\neg Q_h(k_h)$  is the corresponding literal in  $B(P_i) \sigma$ , then the expression  $\neg \exists y B(P_i) \sigma$  can be removed from (26).

If  $\iota Q_h(k_h, u_h)$  is a literal in  $B(P_i^\circ)$  and  $L = \neg Q_h(k_h, v_h)$  is the corresponding literal in  $B(P_i) \sigma$ , then the expression  $\neg \exists y B(P_i) \sigma$  can be simplified to  $\neg \exists y (B(P_i) \sigma \wedge L \wedge u_h \neq v_h)$ .

**Proof:** By (1),  $\iota Q_h(k_h) \rightarrow Q_h(k_h)$  and thus  $\neg Q_h(k_h)$  is false. Also by (1),  $\iota Q_h(k_h, u_h) \rightarrow Q_h(k_h, u_h)$ . Given that  $k_h$  is a key for  $Q_h$ , then  $\neg Q_h(k_h, v_h)$  is true only when  $u_h \neq v_h$ .  $\oplus$

#### Modification of a literal

If  $\mu Q_h(k_h, u_h, u'_h)$  (resp.,  $\mu Q_h(k_h, u'_h, u_h)$ ) is a literal in  $B(P_i^\circ)$  and  $L = Q_h(k_h, v_h)$  (resp.,  $L = \neg Q_h(k_h, v_h)$ ) is the corresponding literal in  $B(P_i) \sigma$ , then the expression  $\neg \exists y B(P_i) \sigma$  can be simplified to  $\neg \exists y (B(P_i) \sigma \wedge L \wedge u'_h = v_h)$  (resp.,  $\neg \exists y (B(P_i) \sigma \wedge L \wedge u_h \neq v_h)$ ).

**Proof:** We give the proof for the modification of a positive literal. By (3),  $\mu Q_h(k_h, u_h, u'_h) \rightarrow Q_h(k_h, u'_h)$ . Given that  $k_h$  is a key for  $Q_h$ , then  $Q_h(k_h, v_h)$  is true only when  $u'_h = v_h$ . We prove similarly the modification of a negative literal.  $\oplus$

#### Unchanged positive literal

If  $Q_h(k_h, u_h)$  is a literal in  $B(P_i^\circ)$  and  $L = Q_h(k_h, v_h)$  is the corresponding literal in  $B(P_i) \sigma$ , then the expression  $\neg \exists y B(P_i) \sigma$  can be simplified to  $\neg \exists y (B(P_i) \sigma \wedge L \wedge u_h \neq v_h)$ . Notice that  $u_h, v_h$  can be null.

**Proof:** Similar to the previous one.  $\oplus$

#### Unchanged negative literal

If  $\neg Q_h(k_h)$  is a literal in  $B(P_i^\circ)$  and  $L = \neg Q_h(k_h)$  is the corresponding literal in  $B(P_i) \sigma$ , then the expression  $\neg \exists y B(P_i) \sigma$  can be simplified to  $\neg \exists y (B(P_i) \sigma \wedge L)$ .

**Proof:** Straightforward.  $\oplus$

#### Auxiliary simplifications

Some of the simplifications insert comparison literals in the expression  $\neg \exists y B(P_i) \sigma$ . A simple analysis of such literals produce further simplifications. For example:

- If a literal has the form  $u = u$ , then it can be removed.
- If a literal has the form  $u \neq u$ , then  $\neg \exists y B(P_i) \sigma$  becomes true.
- If a literal  $L$  has the form  $u_h = v_h$ , where  $u_h$  (resp.,  $v_h$ ) is free (resp., bound) in  $\neg \exists y B(P_i) \sigma$ , then we remove the literal and replace the expression by  $(\neg \exists y B(P_i) \sigma \wedge L) \{u_h/v_h\}$ .

#### Example 4

Applying the above simplifications to the rules corresponding to  $\delta \text{Young}(\underline{p}, a)$  and  $\delta \text{Student}(\underline{p}, a)$  given in Example 3, we get:

- (E.26)  $\delta \text{Young}(\underline{p}, a) \leftarrow \delta \text{Person}(\underline{p}, a) \wedge a < 20$   
(E.27)  $\delta \text{Young}(\underline{p}, a) \leftarrow \mu \text{Person}(\underline{p}, a, a') \wedge a < 20 \wedge \neg a' < 20$   
(E.28)  $\delta \text{Student}(\underline{p}, a) \leftarrow \text{Young}(\underline{p}, a) \wedge \neg \iota \text{Young}(\underline{p}, a)$   
 $\quad \wedge \neg \mu \text{Young}(\underline{p}, a', a) \wedge \iota \text{Works}(\underline{p})$   
(E.29)  $\delta \text{Student}(\underline{p}, a) \leftarrow \delta \text{Young}(\underline{p}, a) \wedge \neg \text{Works}(\underline{p})$   
 $\quad \wedge \neg \delta \text{Works}(\underline{p})$   
(E.30)  $\delta \text{Student}(\underline{p}, a) \leftarrow \delta \text{Young}(\underline{p}, a) \wedge \iota \text{Works}(\underline{p})$   
(E.31)  $\delta \text{Student}(\underline{p}, a) \leftarrow \mu \text{Young}(\underline{p}, a, a') \wedge \iota \text{Works}(\underline{p})$

## 4.2 Simplification of Modification Internal Events Rules

Modification internal events rules for predicate  $P_i$  were defined in section 3.5 as:

- (21)  $\mu P(\underline{k}, \underline{x}, \underline{x}') \leftarrow P_i^\circ(\underline{k}, \underline{x}) \wedge P_h(\underline{k}, \underline{x}') \wedge \underline{x} \neq \underline{x}'$   
 $\quad \quad \quad i=1 \dots m, h=1 \dots m \text{ except } i$   
(22)  $\mu P(\underline{k}, \underline{x}, \underline{x}') \leftarrow \mu P_i(\underline{k}, \underline{x}, \underline{x}') \quad \quad \quad i=1 \dots m$   
(23)  $\mu P_i(\underline{k}, \underline{x}, \underline{x}') \leftarrow P_i^\circ(\underline{k}, \underline{x}) \wedge P_i(\underline{k}, \underline{x}') \wedge \underline{x} \neq \underline{x}'$   
 $\quad \quad \quad i=1 \dots m$

We first show that (21) can be simplified to:

- (27)  $\mu P(\underline{k}, \underline{x}, \underline{x}') \leftarrow \delta P_i(\underline{k}, \underline{x}) \wedge \iota P_h(\underline{k}, \underline{x}') \wedge \underline{x} \neq \underline{x}'$   
 $\quad \quad \quad i=1 \dots m, h=1 \dots m \text{ except } i$

We give the proof in the Appendix. The main idea is that, due to the key integrity constraint,  $P_i(\underline{k}, \underline{x})$  must not hold and that the only way to get  $P_i^\circ(\underline{k}, \underline{x})$  true, being  $P_i(\underline{k}, \underline{x})$  false, is that  $\delta P_i(\underline{k}, \underline{x})$  holds. Also due to the key integrity constraint,  $P_h^\circ(\underline{k}, \underline{x}')$  must not hold and the only way to get  $P_h^\circ(\underline{k}, \underline{x}')$  false, being  $P_h(\underline{k}, \underline{x}')$  true, is that  $\iota P_h(\underline{k}, \underline{x}')$  holds.

Now, we show how to simplify rules (23). Replacing in them  $P_i^\circ(\underline{k}, \underline{x})$  by its equivalent definition given in (11) we get:

- (28)  $\mu P_i(\underline{k}, \underline{x}, \underline{x}') \leftarrow P_i^\circ(\underline{k}, \underline{x}) \wedge P_i(\underline{k}, \underline{x}') \wedge \underline{x} \neq \underline{x}'$   
 $\quad \quad \quad i=1 \dots m, j=1 \dots \alpha$

We can remove from (28) the rules corresponding to  $j=1$ , which have the form given in (12), since  $P_i^\circ(\underline{k}, \underline{x}) \rightarrow P_i(\underline{k}, \underline{x})$  and, due to the key integrity constraint,  $P_i(\underline{k}, \underline{x}) \rightarrow \neg \exists \underline{x}' P_i(\underline{k}, \underline{x}') \wedge \underline{x} \neq \underline{x}'$ . We can then reduce the set (28) to:

- (29)  $\mu P_i(\underline{k}, \underline{x}, \underline{x}') \leftarrow P_i^\circ(\underline{k}, \underline{x}) \wedge P_i(\underline{k}, \underline{x}') \wedge \underline{x} \neq \underline{x}'$   
 $\quad \quad \quad i=1 \dots m, j=2 \dots \alpha$

which can be rewritten as:

- (30)  $\mu P_i(\underline{k}, \underline{x}, \underline{x}') \leftarrow B(P_i^\circ(\underline{k}, \underline{x})) \wedge B(P_i) \wedge \underline{x} \neq \underline{x}'$   
 $\quad \quad \quad i=1 \dots m, j=2 \dots \alpha$

where  $\sigma$  is the substitution  $\{\underline{x}'/\underline{x}, \underline{z}'/\underline{z}\}$ ,  $\underline{z}$  is the set of variables in  $B(P_i)$  except  $\underline{k}$  and  $\underline{x}$ ; and  $\underline{z}'$  are new variables.

As in the deletion case, there are several simplifications that can be applied to rules (29) above. All of them are based on the analysis of the relationship between a literal  $L_h(\underline{k}_h, \underline{u}_h)$  in  $B(P_i^\circ)$  and the corresponding literal in  $B(P_i)\sigma$ . The result of this simplification can be either a reduced form of the expression  $B(P_i)\sigma$  or a removal of the rule.

#### Deletion of a positive literal

If  $\delta Q_h(\underline{k}_h, \underline{u}_h)$  is a literal in  $B(P_i^\circ)$  and  $Q_h(\underline{k}_h, \underline{v}_h)$  is the corresponding literal in  $B(P_i)\sigma$ , then this rule can be removed. Notice that  $\underline{u}_h, \underline{v}_h$  can be null.

**Proof:** By (2),  $\delta Q_h(\underline{k}_h, \underline{u}_h) \rightarrow \neg \exists \underline{y} Q_h(\underline{k}_h, \underline{y})$  and thus  $Q_h(\underline{k}_h, \underline{v}_h)$  is false.  $\oplus$

#### Insertion of a negative literal

If  $\iota Q_h(\underline{k}_h)$  is a literal in  $B(P_i^\circ)$  and  $\neg Q_h(\underline{k}_h)$  is the corresponding literal in  $B(P_i)\sigma$ , then this rule can be removed.

If  $\iota Q_h(\underline{k}_h, \underline{u}_h)$  is a literal in  $B(P_i^\circ)$  and  $L = \neg Q_h(\underline{k}_h, \underline{v}_h)$  is the corresponding literal in  $B(P_i)\sigma$ , then the expression  $B(P_i)\sigma$  can be simplified to  $(B(P_i)\sigma \wedge \underline{u}_h \neq \underline{v}_h)$ .

**Proof:** By (1),  $\iota Q_h(\underline{k}_h) \rightarrow Q_h(\underline{k}_h)$  and thus  $\neg Q_h(\underline{k}_h)$  is false. Also by (1),  $\iota Q_h(\underline{k}_h, \underline{u}_h) \rightarrow Q_h(\underline{k}_h, \underline{u}_h)$ . Given that  $\underline{k}_h$  is a key for  $Q_h$ , then  $\neg Q_h(\underline{k}_h, \underline{v}_h)$  is true only when  $\underline{u}_h \neq \underline{v}_h$ .  $\oplus$

#### Modification of a literal

If  $\mu Q_h(\underline{k}_h, \underline{u}_h, \underline{u}'_h)$  (resp.,  $\mu Q_h(\underline{k}_h, \underline{u}'_h, \underline{u}_h)$ ) is a literal in  $B(P_i^\circ)$  and  $L = Q_h(\underline{k}_h, \underline{v}_h)$  (resp.,  $L = \neg Q_h(\underline{k}_h, \underline{v}_h)$ ) is the corresponding literal in  $B(P_i)\sigma$ , then the expression  $B(P_i)\sigma$  can be simplified to  $(B(P_i)\sigma \wedge \underline{u}'_h = \underline{v}_h)$  (resp.,  $(B(P_i)\sigma \wedge \underline{u}_h \neq \underline{v}_h)$ ).

**Proof:** We give the proof for the modification of a positive literal. By (3),  $\mu Q_h(\underline{k}_h, \underline{u}_h, \underline{u}'_h) \rightarrow Q_h(\underline{k}_h, \underline{u}'_h)$ . Given that  $\underline{k}_h$  is a key for  $Q_h$ , then  $Q_h(\underline{k}_h, \underline{v}_h)$  is true only when  $\underline{u}'_h = \underline{v}_h$ . We prove similarly the modification of a negative literal.  $\oplus$

#### Unchanged positive literal

If  $Q_h(\underline{k}_h, \underline{u}_h)$  is a literal in  $B(P_i^\circ)$  and  $L = Q_h(\underline{k}_h, \underline{v}_h)$  is the corresponding literal in  $B(P_i)\sigma$ , then the expression  $B(P_i)\sigma$  can be simplified to  $(B(P_i)\sigma \wedge \underline{u}_h = \underline{v}_h)$ . Notice that  $\underline{u}_h, \underline{v}_h$  can be null.

**Proof:** Similar to the previous one.  $\oplus$

#### Unchanged negative literal

If  $\neg Q_h(\underline{k}_h)$  is a literal in  $B(P_i^\circ)$  and  $L = \neg Q_h(\underline{k}_h)$  is the corresponding literal in  $B(P_i)\sigma$ , then the expression  $B(P_i)\sigma$  can be simplified to  $(B(P_i)\sigma \wedge L)$ .

**Proof:** Straightforward.  $\oplus$

### Example 5

Applying the above simplifications to the rules corresponding to  $\mu\text{Young}(\underline{p},a,a')$  and  $\mu\text{Student}(\underline{p},a,a')$  given in Example 3, we get:

$$(E.32) \quad \mu\text{Young}(\underline{p},a,a') \leftarrow \mu\text{Person}(\underline{p},a,a') \wedge a < 20 \wedge a' < 20$$

$$(E.33) \quad \mu\text{Student}(\underline{p},a,a') \leftarrow \mu\text{Young}(\underline{p},a,a') \wedge \neg\text{Works}(\underline{p}) \\ \wedge \neg \delta\text{Works}(\underline{p})$$

### 4.3 Simplification of Insertion Internal Events Rules

Insertion internal events rules of predicate  $P_i$  were defined in (15) as:

$$\iota P_i(\underline{k},\underline{x}) \leftarrow P_i(\underline{k},\underline{x}) \wedge \neg \exists y P_i^\circ(\underline{k},\underline{y}) \quad i = 1 \dots m$$

replacing  $P_i^\circ(\underline{k},\underline{y})$  by its equivalent definition given in (11) we get:

$$(31) \quad \iota P_i(\underline{k},\underline{x}) \leftarrow P_i(\underline{k},\underline{x}) \wedge \neg \exists y P_{i,1}^\circ(\underline{k},\underline{y}) \wedge \dots \wedge \\ \neg \exists y P_{i,\alpha}^\circ(\underline{k},\underline{y}) \quad i = 1 \dots m$$

which can be rewritten as:

$$(32) \quad \iota P_i(\underline{k},\underline{x}) \leftarrow B(P_i) \wedge \neg \exists y (B(P_{i,1}^\circ)\sigma) \wedge \dots \wedge \\ \neg \exists y (B(P_{i,\alpha}^\circ)\sigma) \quad i = 1 \dots m$$

where  $\sigma$  is the substitution  $\{y/x, z'/z\}$ ,  $z$  is the set of variables in  $B(P_{i,\alpha}^\circ)$  except  $\underline{k}$  and  $\underline{x}$ ; and  $z'$  are new variables.

Assume  $B(P_i)$  has  $n$  ordinary literals, and let  $L_h = [Q_h(\underline{k}_h, \underline{u}_h) \mid \neg Q_h(\underline{k}_h, \underline{u}_h)]$  be one of them. It is not difficult to see that in (32) an  $\iota P_i$  fact can only be induced by an insertion, deletion or modification event of some  $L_h$ . In fact, we prove in [Urp91a] that each rule (32) is equivalent to the set of rules:

$$(33) \quad \iota P_i(\underline{k},\underline{x}) \leftarrow B(P_i) \setminus L_h \\ \wedge [\iota Q_h(\underline{k}_h, \underline{u}_h) \mid \delta Q_h(\underline{k}_h, \underline{u}_h)] \\ \wedge \neg \exists y (B(P_{i,1}^\circ)\sigma) \wedge \dots \wedge \neg \exists y (B(P_{i,\alpha}^\circ)\sigma) \\ i = 1 \dots m, h = 1 \dots n$$

$$(34) \quad \iota P_i(\underline{k},\underline{x}) \leftarrow B(P_i) \setminus L_h \\ \wedge [\mu Q_h(\underline{k}_h, \underline{u}'_h, \underline{u}_h) \mid \mu Q_h(\underline{k}_h, \underline{u}_h, \underline{u}'_h)] \\ \wedge \neg \exists y (B(P_{i,1}^\circ)\sigma) \wedge \dots \wedge \neg \exists y (B(P_{i,\alpha}^\circ)\sigma) \\ i = 1 \dots m, h = 1 \dots n$$

As in the previous case, there are several simplifications that can be applied to rules (33) and (34) above. Again, these simplifications are based on the analysis of the relationship between a literal in  $B(P_i) \setminus L_h$  or  $\iota Q_h$ ,  $\delta Q_h$ ,  $\mu Q_h$  literals and the corresponding literal in each  $B(P_{i,j}^\circ)\sigma$ , where  $j = 1 \dots \alpha$ . The result of this simplification can be either a reduced form of the expressions  $\neg \exists y B(P_{i,j}^\circ)\sigma$  or the removal of whole rule.

#### Insertion of a positive literal

If  $\iota Q_h(\underline{k}_h, \underline{u}_h)$  is a literal in (33) and the key of the corresponding literals of some  $B(P_{i,j}^\circ)\sigma$  is  $\underline{k}_h$ , then the expression  $\neg \exists y (B(P_{i,j}^\circ)\sigma)$  can be removed from (33). Notice that  $\underline{u}_h$  can be null.

**Proof:** The corresponding literals of  $\iota Q_h(\underline{k}_h, \underline{u}_h)$  in  $B(P_{i,j}^\circ)\sigma$  have one of the form  $Q_h(\underline{k}_h, \underline{v}_h) \wedge \neg \iota Q_h(\underline{k}_h, \underline{v}_h) \wedge \neg \mu Q_h(\underline{k}_h, \underline{v}'_h, \underline{v}_h)$  or  $\delta Q_h(\underline{k}_h, \underline{v}_h)$  or  $\mu Q_h(\underline{k}_h, \underline{v}_h, \underline{v}'_h)$ . Assume the first form. Then, by (1),  $\iota Q_h(\underline{k}_h, \underline{u}_h) \rightarrow Q_h(\underline{k}_h, \underline{u}_h)$ . Given that  $\underline{k}_h$  is a key for  $Q_h$ , then  $Q_h(\underline{k}_h, \underline{v}_h)$  is true only when  $\underline{u}_h = \underline{v}_h$ . But in this case,  $\neg \iota Q_h(\underline{k}_h, \underline{v}_h)$  is false.

For the other two forms, we have  $\iota Q_h(\underline{k}_h, \underline{u}_h) \rightarrow \neg \exists y \delta Q_h(\underline{k}_h, \underline{y})$  and  $\iota Q_h(\underline{k}_h, \underline{u}_h) \rightarrow \neg \exists y, y' \mu Q_h(\underline{k}_h, \underline{y}, \underline{y}')$ .  $\oplus$

#### Deletion of a negative literal

If  $\delta Q_h(\underline{k}_h)$  is a literal in (33) and  $\neg Q_h(\underline{k}_h) \wedge \neg \delta Q_h(\underline{k}_h)$  or  $\iota Q_h(\underline{k}_h)$  is the corresponding literal in some  $B(P_{i,j}^\circ)\sigma$ , then the expression  $\neg \exists y (B(P_{i,j}^\circ)\sigma)$  can be removed from (33).

If  $\delta Q_h(\underline{k}_h, \underline{u}_h)$  is a literal in (33) and  $\iota Q_h(\underline{k}_h, \underline{v}_h)$  or  $\mu Q_h(\underline{k}_h, \underline{v}'_h, \underline{v}_h)$  is the corresponding literal in some  $B(P_{i,j}^\circ)\sigma$ , then the expression  $\neg \exists y (B(P_{i,j}^\circ)\sigma)$  can be removed from (33).

If  $\delta Q_h(\underline{k}_h, \underline{u}_h)$  is a literal in (33) and  $L = \neg Q_h(\underline{k}_h, \underline{v}_h) \wedge \neg \delta Q_h(\underline{k}_h, \underline{v}_h) \wedge \neg \mu Q_h(\underline{k}_h, \underline{v}_h, \underline{v}'_h)$  is the corresponding literal in some  $B(P_{i,j}^\circ)\sigma$ , then the expression  $\neg \exists y (B(P_{i,j}^\circ)\sigma)$  can be simplified to  $\neg \exists y (B(P_{i,j}^\circ)\sigma) \wedge L$  ( $\underline{u}_h \neq \underline{v}_h$ ).

**Proof:** By (2),  $\delta Q_h(\underline{k}_h, \underline{u}_h) \rightarrow \neg \exists y Q_h(\underline{k}_h, \underline{y})$ . Given that  $\underline{k}_h$  is a key for  $Q_h$ , then  $\neg \delta Q_h(\underline{k}_h, \underline{v}_h)$  is true only when  $\underline{u}_h \neq \underline{v}_h$ . Finally,  $\delta Q_h(\underline{k}_h, \underline{u}_h) \rightarrow \neg \exists y \iota Q_h(\underline{k}_h, \underline{y})$  and  $\delta Q_h(\underline{k}_h, \underline{u}_h) \rightarrow \neg \exists y, y' \mu Q_h(\underline{k}_h, \underline{y}, \underline{y}')$ .  $\oplus$

#### Modification of a positive literal

If  $\mu Q_h(\underline{k}_h, \underline{u}'_h, \underline{u}_h)$  is a literal in (34) and  $Q_h(\underline{k}_h, \underline{v}_h) \wedge \neg \iota Q_h(\underline{k}_h, \underline{v}_h) \wedge \neg \mu Q_h(\underline{k}_h, \underline{v}'_h, \underline{v}_h)$  or  $\delta Q_h(\underline{k}_h, \underline{v}_h)$  is the corresponding literal in some  $B(P_{i,j}^\circ)\sigma$ , then the expression  $\neg \exists y (B(P_{i,j}^\circ)\sigma)$  can be removed from (34).

If  $\mu Q_h(\underline{k}_h, \underline{u}'_h, \underline{u}_h)$  is a literal in (34) and  $L = \mu Q_h(\underline{k}_h, \underline{v}_h, \underline{v}'_h)$  is the corresponding literal in some  $B(P_{i,j}^\circ)\sigma$ , then the expression  $\neg \exists y (B(P_{i,j}^\circ)\sigma)$  can be simplified to  $\neg \exists y (B(P_{i,j}^\circ)\sigma) \wedge L$  ( $\underline{u}'_h = \underline{v}_h \wedge \underline{u}_h = \underline{v}'_h$ ).

**Proof:** By (3),  $\mu Q_h(\underline{k}_h, \underline{u}'_h, \underline{u}_h) \rightarrow Q_h(\underline{k}_h, \underline{u}_h)$ . Given that  $\underline{k}_h$  is a key for  $Q_h$ , then  $Q_h(\underline{k}_h, \underline{v}_h)$  is true only when  $\underline{u}_h = \underline{v}_h$ . But in this case,  $\neg \mu Q_h(\underline{k}_h, \underline{v}'_h, \underline{v}_h)$  is false. Furthermore,  $\mu Q_h(\underline{k}_h, \underline{u}'_h, \underline{u}_h) \rightarrow \neg \exists y \delta Q_h(\underline{k}_h, \underline{y})$ .

On the other hand, given that  $\underline{k}_h$  is a key for  $Q_h$ , then  $\mu Q_h(\underline{k}_h, \underline{v}_h, \underline{v}'_h)$  is true only when  $\underline{u}'_h = \underline{v}_h \wedge \underline{u}_h = \underline{v}'_h$ .  $\oplus$

#### Modification of a negative literal

If  $\mu Q_h(\underline{k}_h, \underline{u}_h, \underline{u}'_h)$  is a literal in (34) and  $\iota Q_h(\underline{k}_h, \underline{v}_h)$  is the corresponding literal in some  $B(P_{i,j}^\circ)\sigma$ , then the expression  $\neg \exists y (B(P_{i,j}^\circ)\sigma)$  can be removed from (34).



If  $\mu Q_h(k_h, u_h, u'_h)$  is a literal in (34) and  $L = -Q_h(k_h, v_h) \wedge \neg \delta Q_h(k_h, v_h) \wedge \neg \mu Q_h(k_h, v_h, v'_h)$  (resp.,  $L = \mu Q_h(k_h, v_h, v'_h)$ ) is the corresponding literal in some  $B(P_{ij}^\circ)\sigma$ , then the expression  $\neg \exists y(B(P_{ij}^\circ)\sigma)$  can be simplified to  $\neg \exists y(B(P_{ij}^\circ)\sigma) \wedge u'_h \neq v_h \wedge u_h \neq v_h$  (resp.,  $\neg \exists y(B(P_{ij}^\circ)\sigma) \wedge u_h = v'_h \wedge u'_h = v_h$ ).

**Proof:** Similar to previous one.  $\oplus$

#### Unchanged positive literal

If  $Q_h(k_h, u_h)$  is a literal in (33) (or (34)) and  $\delta Q_h(k_h, v_h)$  is the corresponding literal in some  $B(P_{ij}^\circ)\sigma$ , then the expression  $\neg \exists y(B(P_{ij}^\circ)\sigma)$  can be removed from (33) (from (34)).

If  $Q_h(k_h, u_h)$  is a literal in (33) (or (34)) and  $L = Q_h(k_h, v_h) \wedge \neg \iota Q_h(k_h, v_h) \wedge \neg \mu Q_h(k_h, v_h, v'_h)$  (resp.,  $L = \mu Q_h(k_h, v_h, v'_h)$ ) is the corresponding literal in some  $B(P_{ij}^\circ)\sigma$ , then the expression  $\neg \exists y(B(P_{ij}^\circ)\sigma)$  can be simplified to  $\neg \exists y(B(P_{ij}^\circ)\sigma) \wedge u_h = v_h$  (resp.,  $\neg \exists y(B(P_{ij}^\circ)\sigma) \wedge u_h = v'_h$ ). Notice that  $u_h, v_h$  can be null.

**Proof:** We prove the first case. By the sake of a contradiction, if  $\delta Q_h(k_h, v_h)$  was true, then, by (2),  $\neg \exists y Q_h(k_h, y)$ . But, since  $Q_h(k_h, u_h)$  is a literal in (33) (or (34)), we have a contradiction.  $\oplus$

#### Unchanged negative literal

If  $-Q_h(k_h)$  is a literal in (33) (or (34)) and  $\iota Q_h(k_h)$  is the corresponding literal in some  $B(P_{ij}^\circ)\sigma$ , then the expression  $\neg \exists y(B(P_{ij}^\circ)\sigma)$  can be removed from (33) (from (34)).

If  $-Q_h(k_h)$  is a literal in (33) (or (34)) and  $L = -Q_h(k_h) \wedge \neg \delta Q_h(k_h)$  is the corresponding literal in some  $B(P_{ij}^\circ)\sigma$ , then the expression  $\neg \exists y(B(P_{ij}^\circ)\sigma)$  can be simplified to  $\neg \exists y(B(P_{ij}^\circ)\sigma)$ .

**Proof:** Similar to the previous one.  $\oplus$

### Example 6

Applying the above simplifications to the rules corresponding to  $\iota \text{Young}(p, a)$  and  $\iota \text{Student}(p, a)$  given in Example 3, we obtain:

$$(E.34) \quad \iota \text{Young}(p, a) \leftarrow \iota \text{Person}(p, a) \wedge a < 20$$

$$(E.35) \quad \iota \text{Young}(p, a) \leftarrow \mu \text{Person}(p, a', a) \wedge a < 20 \wedge \neg a' < 20$$

$$(E.36) \quad \iota \text{Student}(p, a) \leftarrow \iota \text{Young}(p, a) \wedge \neg \text{Works}(p)$$

$$(E.37) \quad \iota \text{Student}(p, a) \leftarrow \text{Young}(p, a) \wedge \delta \text{Works}(p)$$

## 5 Change Computation

We present in this Section a method for the definition and computation of changes in deductive databases. Efficient change computation is essential in a wide range of applications in deductive databases, including integrity constraints checking, view materialization and condition monitoring. The common pattern in all these applications consists of:

- The definition of one or more changes to be monitored.
- The computation of the changes induced by a database update.
- The execution of some action when some of the defined changes has been induced.

We will show first that our internal event concept can be used to define the changes to be monitored. Assume that  $Ic$  is an inconsistency predicate, such as, for example:

$$Ic(e, c) \leftarrow \text{Works}(e, c) \wedge \neg \text{Company}(c)$$

meaning that employees must work in companies. Then, insertion internal events  $\iota Ic$  will represent violations of the corresponding integrity constraint. If an update to base predicates induces some  $\iota Ic$  fact then the update must be rejected. Deletion and modification internal events are not defined for inconsistency predicates, since we assume that the database is consistent before the update and, therefore, predicate  $Ic$  is false.

Now, assume that  $Em$  is a derived predicate corresponding to a materialized view, such as, for example:

$$Em(\text{emp}, \text{manager}) \leftarrow Ed(\text{emp}, \text{dept}) \wedge Dm(\text{dept}, \text{manager})$$

In this case, internal events  $\iota Em$ ,  $\delta Em$  and  $\mu Em$  correspond to the insertion, deletion or modification of facts in the extension of  $Em$ . Thus, for instance, if the update induces an  $\iota Em(E, M)$  fact, then  $Em(E, M)$  will be inserted into the extension of the materialized view  $Em$ .

General conditions can also be represented as insertion, deletion or modification internal events of a derived predicate. Assume, for example, that we want to monitor changes of employees earning more than 1000. We may define a predicate  $C$ :

$$C(\text{emp}, \text{salary}) \leftarrow \text{Sal}(\text{emp}, \text{salary}) \wedge \text{salary} > 1000$$

and then  $\iota C(e, s)$  can be used to define a change meaning that  $e$  is an employee earning more than 1000 after the update, but not before;  $\delta C(e, s)$  for a change meaning that employee  $e$  ceases to earn more than 1000; and  $\mu C(e, s, s')$  for a change meaning that the salary of employee  $e$  has been modified from  $s$  to  $s'$ , both greater than 1000. Appropriate actions could be associated to each, or some, of the above changes.

Thus, we see that the single concept of internal event may serve for defining relevant changes in a variety of applications in deductive databases.

### 5.1 Our Method

We now describe our method for change computation. The method can be entirely based on the use of standard SLDNF resolution. Let  $D$  be a deductive database and let



with more than one rule (with the binary union operator as a root of the expression).

Furthermore, our rules incorporate the knowledge of keys of predicates. This allow us to obtain a more simplified set of rules that fit to each particular situation. As an example, consider the derived predicate  $Young(p,a)$ , defined in the example 1 as:  $Young(p,a) \leftarrow Person(p,a) \wedge a < 20$ . We have shown that rules E.26, E.27, E.32, E.34 and E.36 compute changes to that predicate in incremental form. Assuming now that our knowledge of keys changes:  $Young(p,a) \leftarrow Person(p,a) \wedge a < 20$ , applying our method we obtain the following simple events rules:

$$\begin{aligned} \iota Young(p,a) &\leftarrow \iota Person(p,a) \wedge a < 20 \\ \delta Young(p,a) &\leftarrow \delta Person(p,a) \wedge a < 20 \end{aligned}$$

For a more detailed comparison see [Urp91b].

## 6.2 Ceri and Widom's Method

This method derives automatically production rules for incremental maintenance of materialized views. The rules are executable using the rule language of the Starburst database system [WCL91]. Views are specified using a standard query language, and arbitrary database updates (insertions, deletions and/or modifications) are considered. The method defines insertion and deletion changes of a materialized view. These changes are computed once base relations have been updated. Using the definition of a view and information about keys of the view's base tables, the method determines whether efficient view maintenance production rules [WiF90, WCL91] for updates on each base table can be generated.

If a base table reference in a view definition is *safe* [CeW91], incremental view maintenance rules can be generated. However, if a base table reference is *unsafe*, some of the updates on this table cannot be handled in incremental form and a rule that rematerializes the view in such cases is defined.

Once the rules have been generated, they must be ordered using a *precede* clause so that all rules performing deletions precede all rules performing insertions. The rematerialization rule of a view (if exists) has precedence over all rules of that view.

In general, using our method, we obtain similar results when a base table reference is *safe*. But there are two main differences. The first is that we can induce not only insertion and deletion changes of a materialized view, but also modification changes. To see the importance of this extension consider the derived predicate  $Young(p,a)$  given in the Example 1 and assume that  $Young$  is a materialized view. Using our method, we get the following simplified events rules that handle modifications events of base predicate  $Person$ :

$$(E.27) \delta Young(p,a) \leftarrow \mu Person(p,a,a') \wedge a < 20 \wedge \neg a' < 20$$

$$(E.32) \mu Young(p,a,a') \leftarrow \mu Person(p,a,a') \wedge a < 20 \wedge a' < 20$$

$$(E.35) \iota Young(p,a) \leftarrow \mu Person(p,a',a) \wedge a < 20 \wedge \neg a' < 20$$

The first rule can induce deletion changes of predicate  $young$ ; the second one, modification changes and the third one, insertion changes. Instead, rules generated using Ceri and Widom's method handle case E.32 in a more inefficient way: as deletions of tuples  $Young(p,a)$  followed by insertions of tuples  $Young(p,a')$ .

A second difference is that rules generated using Ceri and Widom's method do unnecessary work when the base table attributes that are irrelevant to the view definition are modified. As an example, consider the derived predicate  $Employee(e)$ , defined as  $Employee(e) \leftarrow Works(e,c)$ . Applying our method we get the simplified events rules:

$$\begin{aligned} \iota Employee(e) &\leftarrow \iota Works(e,c) \\ \delta Employee(e) &\leftarrow \delta Works(e,c) \end{aligned}$$

Notice that, since a modification of the company in which an employee works cannot induce insertion or deletion changes on  $Employee$ , our rules do not take those modifications into account. However, rules generated using Ceri and Widom's method do unnecessary work since such modifications will be handled as a deletion of tuple  $Employee(e)$  followed by an insertion of the same tuple.

As we mentioned previously, using Ceri and Widom's method, when a base table reference in a view definition is *unsafe*, some of the updates on that table cannot be supported efficiently. In this case, the main advantage of our method relies on the fact that we can handle those updates in incremental form. As an example, consider the derived predicate  $Tenant(p,h)$ , defined as  $Tenant(p,h) \leftarrow Lives(p,h) \wedge Owns(c,h)$ . Since  $Owns(c,h)$  is *unsafe*, if a tuple is modified or deleted from this table, the view  $Tenant(p,h)$  will be rematerialized. Instead, applying our method we get the following set of simplified rules that handle deletions from  $Owns(c,h)$  in incremental form:

$$\begin{aligned} \delta Tenant(p,h) &\leftarrow Lives(p,h) \wedge \neg \iota Lives(p,h) \\ &\quad \wedge \neg \mu Lives(p,h',h) \wedge \delta Owns(c,h) \\ &\quad \wedge \neg Owns(c',h) \\ \delta Tenant(p,h) &\leftarrow \delta Lives(p,h) \wedge \delta Owns(c,h) \\ \delta Tenant(p,h) &\leftarrow \mu Lives(p,h,h') \wedge \delta Owns(c,h) \\ &\quad \wedge \neg Owns(c',h') \\ \mu Tenant(p,h,h') &\leftarrow \mu Lives(p,h,h') \wedge \delta Owns(c,h) \\ &\quad \wedge Owns(c',h') \end{aligned}$$

Similar rules are obtained for modifications of  $Owns(c,h)$ .

Finally, we point out that our rules are generated using a simple procedure, while Ceri and Widom's rules are generated using a complex procedure, that needs to take into account the potentially complex syntactic structure of the view definition.

## 7 Conclusions

In this paper, we have presented a formal method to derive a set of transition and internal events rules for a deductive database. Given an update, the transition rules relate the old state to the new state and the events induced by the update. The internal events rules define explicitly the changes (insertions, deletions and modifications) induced by the update on the derived predicates.

We have then presented a method that use the above rules for computing the changes induced by an update in a deductive database. The method deals with allowed, stratified databases. Updates considered are sets of insertions, deletions and/or modifications of base facts. Our method is based on the use of the standard SLDNF procedure and, in this way, it can be implemented directly in Prolog. However, other proof procedures could be used as well. Some optimization techniques, including partial evaluation, can be easily incorporated into our method.

We have also compared our method with some other well-known methods, and we have shown how we improve their efficiency.

We plan to further simplify our transition and internal events rules by taking into account the complete set of integrity constraints of the database, including alternate keys.

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## Appendix

**Proof of (27):** We prove this simplification in two steps. Firstly, we prove that, due to the key integrity constraint,  $P_i(k,x)$  must not hold and that the only way to get  $P_i(k,x)$  true, being  $P_i(k,x)$  false, is that  $\delta P_i(k,x)$  holds. Secondly, we prove that, due to the key integrity constraint,  $P_h^\circ(k,x')$  must not hold and the only way to get  $P_h^\circ(k,x')$  false, being  $P_h(k,x')$  true, is that  $\iota P_h(k,x')$  holds.

*First step.*

Replacing  $P_i^\circ(k,x)$  by its definition given in (4) we rewrite (21) into the set of rules:

$$(P.1) \mu P(k,x,x') \leftarrow P_i(k,x) \wedge \neg \iota P_i(k,x)$$

$$\wedge \neg \mu P_i(k,x'',x) \wedge P_h(k,x') \wedge x \neq x'$$

$$(P.2) \mu P(k,x,x') \leftarrow \delta P_i(k,x) \wedge P_h(k,x') \wedge x \neq x'$$

$$(P.3) \mu P(k,x,x') \leftarrow \mu P_i(k,x,x'') \wedge P_h(k,x') \wedge x \neq x'$$

where  $i = 1 \dots m, h = 1 \dots m$  except  $i$ .

Rules (P.1) above can be removed since that, by the

key integrity constraint,  $P_i(k,x) \rightarrow \neg \exists x' P_h(k,x') \wedge x \neq x'$ .

Since, by (3),  $\mu P_i(k,x,x'') \rightarrow P_i(k,x'')$  and given that  $k$  is a key for  $P$ , then  $x'' = x'$  and thus rules (P.3) above can be removed since they are subsumed by (22).

*Second step.*

Let us consider rules (P.2). Since we assume that key integrity constraints hold before and after the update, we can rewrite rules (P.2) as:

$$(P.4) \mu P(k,x,x') \leftarrow \delta P_i(k,x) \wedge P_h(k,x') \wedge x \neq x' \\ \wedge \neg (P_i^\circ(k,x) \wedge P_h^\circ(k,x') \wedge x \neq x')$$

and given that, by (2),  $\delta P_i(k,x) \rightarrow P_i^\circ(k,x)$ , we can rewrite (P.4) as:

$$(P.5) \mu P(k,x,x') \leftarrow \delta P_i(k,x) \wedge P_h(k,x') \wedge x \neq x' \\ \wedge \neg P_h^\circ(k,x').$$

Replacing  $\neg P_h^\circ(k,x')$  by its definition given in (5) we transform (P.5) into the set of rules:

$$(P.6) \mu P(k,x,x') \leftarrow \delta P_i(k,x) \wedge P_h(k,x') \wedge x \neq x' \\ \wedge \neg P_h(k,x') \wedge \neg \delta P_h(k,x') \wedge \neg \mu P_h(k,x',x'')$$

$$(P.7) \mu P(k,x,x') \leftarrow \delta P_i(k,x) \wedge P_h(k,x') \wedge x \neq x' \\ \wedge \iota P_h(k,x')$$

$$(P.8) \mu P(k,x,x') \leftarrow \delta P_i(k,x) \wedge P_h(k,x') \wedge x \neq x' \\ \wedge \mu P_h(k,x'',x')$$

where  $i = 1 \dots m, h = 1 \dots m$  except  $i$ .

Rules (P.6) above can be removed since  $P_h(k,x') \wedge \neg P_h(k,x')$  can not hold.

Given that, by (1),  $\iota P_h(k,x') \rightarrow P_h(k,x')$  we transform rules (P.7) in (30).

Since, by (2),  $\delta P_i(k,x) \rightarrow P_i^\circ(k,x)$ , by (3),  $\mu P_h(k,x'',x') \rightarrow P_h^\circ(k,x'')$  and given that  $k$  is key for  $P$ , then  $x=x''$  and thus rules (P.8) can also be removed since they are subsumed by (22).  $\oplus$

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