A Domain-theoretic Approach to Integrating Functional and Logic Database Languages

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Abstract

The advantages of logic languages with respect to search-based computation are well-understood, while the advantages of functional languages with respect to deterministic computation are becoming increasingly recognised. It is therefore natural to investigate the development of languages which reconcile the two paradigms. As a contribution to this effort, we extend an existing functional database language called PFL with sets as first class objects. The resulting language subsumes $\text{Datalog}^{\text{fun+neg}}$ in the sense that any set of $\text{Datalog}^{\text{fun+neg}}$ rules can be translated into a set of PFL equations with the same semantics. Since functional and logic database languages can be considered as proper sub-languages of PFL, well-known optimisation techniques from both can usefully be employed (for example lazy evaluation for recursive functions and bottom-up evaluation techniques for recursive predicates).

We motivate our work by reviewing the respective advantages of functional and logic programming for computation, data manipulation and data modelling. An overview of the previous version of PFL is presented and the syntax of this language is then extended to incorporate sets. We show how the Plotkin powerdomain construction can be used to assign meaning to set expressions and we give a denotational semantics for the extended language. To illustrate its expressiveness, we show how Datalog rules can be expressed as PFL functions. We discuss the optimisation of these functions. We also show how integrity constraints can be defined, and describe how a particular constraint enforcement technique developed for logic databases can be adopted by PFL.

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1. Introduction

Recently we have been investigating the implementation of deductive databases whose inference is based on functional programming rather than the more common approach of logic programming. In particular, we introduced in [29, 33] a functional database language called PFL which supports higher-order functions, recursive data types, and the persistence of factual and procedural information, both in the form of equations. A unique feature of PFL is that bulk data is stored using a class of extensionally-defined updateable functions called *selectors*. Selectors are invertible, in the sense that they simulate extensional predicates. We showed in [29] how functions can be written which draw inferences from selectors and which are also invertible. In particular, we examined the expressiveness of these invertible functions and showed how any stratified Datalog IDB predicate can be simulated by a such a function, subject to the proviso that there exists a pre-determined order of firing the rules which define the predicate.

The main aim of the present paper is to remove this restriction and hence to endow PFL's invertible functions with at least the full expressiveness of Datalog. The result is a language which combines the advantages of functional programming with respect to deterministic computations with the advantages of logic programming with respect to search-based computations. This was not achieved in the original language since in general rules can be defined for which no fixed firing sequence generates all the provable facts. An example of such a set of rules is the following (from [22]) :

p(a,b) ←
p(c,b) ←
p(X,Z) ← p(X,Y), p(Y,Z)
p(X,Y) ← p(Y,X)

p(a,c) is provable from these rules, but not by any depth-first search strategy with a fixed ordering of trying the rules, since the last two rules have equally general heads.

In order to attain at least the expressiveness of Datalog, we introduce *sets* into PFL. This allows us to express rules without any implied order of firing. Our sets are first class objects : they can appear arbitrarily nested within expressions and can themselves contain arbitrarily nested expressions; also, functions can be written which take sets as arguments and return sets as results. In particular, functions can be defined over sets to simulate all Datalog$^{fun+neg}$ predicates. We term such functions *intentional selectors*. As we would expect, the new PFL is in fact more expressive than Datalog since aggregation and counting operations can be defined over sets.

A secondary aim of the paper is to show how semantic integrity constraints can be defined and efficiently enforced in the extended PFL. An immediate consequence of supporting sets as first class objects is that approaches developed for the efficient enforcement of constraints in logic databases can be adopted. As an example of this we illustrate how the method of Lloyd and Topor [21] can be used. In fact, our integrity constraints are more expressive than those of logic databases since cardinality and other aggregation constraints are supported.

Since PFL aims to combine the respective advantages of logic and functional programming, we briefly review these in the remainder of this introductory section.

One major advantage of functional languages over logic languages is that they are higher order and so all expressions are first class objects. Higher-order functions can be defined which abstract out recursion patterns over recursive data structures, and which can then be used in the definition of other functions which do not include explicit recursion. A second advantage is that the deterministic semantics of functional evaluation can be exploited for the representation of *default rules* (see [28]). In contrast, logic languages are monotonic and hence require the introduction of extra-logical constructs such as the negation as failure rule [14]. Thirdly, by employing lazy evaluation, infinite processes and data structures can be handled. Finally, deterministic computations are expressed more succinctly by nested function applications than by joins of predicates over common variables.

Conversely, the main advantage of logic languages is that search-based computations are supported directly, while in functional languages they must be simulated deterministically. Also, unlike functions, predicates are invertible in the sense that they can, theoretically, be used with any of their arguments uninstantiated, and thus predicates may be more versatile than functions. Finally, in the context of deductive databases both facts and derivation rules are often expressed more naturally in terms of single predicates than in terms of functions and their inverses.

The motivation behind the development of PFL has been to combine within one uniform semantic and operational framework the respective advantages of these two paradigms for deductive databases. In particular, the extended PFL exhibits all of the features of functional languages described above. Furthermore, since sets are now first class objects, functions can be defined which play a role similar to intentional predicates. Any set of Datalog rules (including those incorporating negation and function symbols) can be translated into an equivalent set of PFL equations. Finally, all of the query optimisation strategies developed for logic languages - including optimisation techniques developed for integrity constraints - can now be transferred to a functional context.

The outline of the remainder of the paper is as follows. In section 2 we briefly describe the features of our original language, and in particular its type system and the way in which user defined functions are specified. Section 3 introduces the syntax and semantics of the extended PFL. We first define a kernel functional language and give the denotational semantics of this language. We then add set expressions and extend the denotational semantics accordingly. Finally, we describe how bulk data is stored in a class of extensionally-defined set-valued functions. Section 4 then discusses the evaluation of expressions in the extended PFL. Section 5 considers the expressiveness of the language. We give a scheme for translating rules in Datalog$^{fun+neg}$ into equations involving set expressions, and then give an optimisation scheme for these equations. We also show how higher order functions can be applied to sets, thus allowing computations to be defined which would require extra-logical predicates such as "set-of" in a logic language. Finally, we discuss the definition and efficient enforcement of integrity constraints. Section 6 gives our conclusions and a brief comparison with related work.

2. Overview of PFL

PFL has a Milner-style polymorphic, strong, static type system [10] that supports a number of built-in types and provides facilities for the introduction of new user-defined types and constants. Functions are defined incrementally through the insertion and deletion of equations. The types of functions are inferred incrementally in the face of such updates. Function evaluation is lazy, thereby permitting computation with unbounded data structures. Types, constants and equations can be defined and removed at any stage during the life-time of the database subject to certain obvious restrictions (e.g. a constant cannot be introduced for an as-yet unspecified type). All types, constants and equations are stored in the database. We briefly review the structure of the type system in 2.1, the system defined functions in 2.2, and the specification of user defined functions in 2.3.

2.1 The Type System

PFL's type system comprises three layers (c.f. [11]). Firstly, there is the meta-level type Type which is the set of all types; secondly, there are the object-level types, both built-in and user-declared, which are regarded as meta-level values of type Type; and finally there are the values corresponding to each object-level type. The built-in types are Bool, Num, Str, and Char which are populated by booleans, strings, numbers and characters respectively. In addition, for every type t there is a distinguished constant Any_t. We usually write this constant without its subscript since this can be inferred from context. Certain system-defined functions when applied to Any will result in a run-time error e.g. the arithmetic functions (c.f. division by zero).

New types and values are declared using commands of the form

"type" constant type-variables ";"
"value" constant "::" type ";"

These constants are sometimes termed *constructors* [27] since they construct values of the indicated types. For example, Person, Product and List types together with constructors for these types are defined as follows, where identifiers starting with an uppercase letter are constructors (meta-level or object-level), whilst identifiers starting with a lowercase letter are variables :

```
type   Person;
type   Prod2 a b;
type   List a;
value  John ::     Person;
value  Tuple2 ::   a→b→(Prod2 a b);
value  (:) ::      a→(List a)→(List a);
value  [] ::       List a;
```

Thus, Person is a type (or meta-level value), Prod2 t1 t2 is a type for all types t1 and t2, and List t is a type for all types t. Similarly, John is a value of type Person, and Tuple2 v1 v2 is a value of type Prod2 t1 t2, assuming v1 and v2 are of type t1 and t2 respectively. Zero argument types (e.g. Person) and values (e.g. John) can be regarded as object types and object values respectively. For syntactic ease we adopt a number of abbreviations for types and values :

```
List t                ≡   [t]
Prodn t1 ... tn       ≡   (t1,...,tn)
(:) v1 v2             ≡   (v1:v2)
v1:(...(vn:[])...)    ≡   [v1,...,vn]
Tuplen v1 ... vn      ≡   (v1,...,vn)
```

In preparation for adding sets to PFL, we also assume the availability of a polymorphic set type Set a, and we allow the syntax {t} in lieu of Set t for any type expression t (analogously to [t] for List t).

2.2 System Defined Functions

PFL provides a number of system defined functions, including the usual arithmetic (+, −, *, /, div, mod), relational (=, !=, <, >, ≥, ≤, and, or, not) and list processing operators (++, head, tail). In addition, the operator ?= (pronounced "matches") acts on values containing Any. In particular, ?= determines whether its left operand is identical to its right operand except with respect to occurrences of Any in the former. More precisely, for each n-ary constructor C we have that

```
(C x1 ... xn) ?= (C y1 ... yn)  =
     True and
     (x1 ?= y1) and ... and (xn ?= yn)
```

while for any other values x and y we have that

```
x ?= y    =    x = Any
```

(we observe here that the first = symbol in a PFL equation denotes definition while thereafter = denotes equality). For example, [Any,2,Any] ?= [1,2,3] and [1,2,3] ?= [Any,2,Any] evaluate to True and False respectively. We note that ?= is reflexive, anti-symmetric and transitive i.e. it is a partial ordering.

2.3 User Defined Functions

Functions are defined by means of equations, which are introduced by a command of the form

"define" lhs "=" expr ";"

For example the 3-ary function if and the 2-ary function map (which has the property that map f $[x_1,...,x_n]$ yields $[f\ x_1, ..., f\ x_n]$) can be defined as follows :

```
define if True x y  =  x;
define if False x y =  y;
define map f []     =  [];
define map f (x:y)  =  (f x):(map f y);
```

The types of these functions are inferred automatically by the system.

A useful query construct supported by PFL, as by most functional languages, is the *list abstraction*, which is simply syntactic sugar for ordinary function applications (see [27]). An example of the use of the list abstraction is the quick-sort function below (where ++ is the infix append operation) :

```
define qsort []    = [];
define qsort (h:t) =
     (qsort [x | x ← t ; x < h]) ++
     [h] ++
     (qsort [x | x ← t ; x ≥ h]);
```

3. Extending PFL with Sets

In this section we describe how PFL is extended with sets as first-class objects, and we specify formally the semantics of the resulting language. The formal foundation of any functional language is the λ calculus (of which a detailed account may be found in [19]). Computation in this calculus consists of rewriting expressions to a normal form by a series of syntactic transformations called β and η reductions. A formal theory can be constructed for βη reduction and models of this theory are triples <Λ, •, D>, where D is a domain of continuous functions (we define the terms *domain* and *continuous* in 3.1 below), Λ models the definition of functions by λ abstraction and • models function application. The semantics of functional languages are thus usually specified using a *denotational* approach [31, 35] which assigns values in D to expressions in the syntax of the language. We too take this route for specifying the semantics of PFL. We begin by reviewing the prerequisite basic domain theory in 3.1. In 3.2 we define the syntax and denotational semantics of a kernel functional language into which PFL expressions are translated before evaluation. In 3.3 we add set expressions to this syntax and in 3.4 we extend the denotational semantics accordingly. In 3.5 we extend the kernel further to allow pattern-matching. Finally, in 3.6 we discuss a class of set-valued functions called *selectors* which are used for the storage and update of bulk data.

Readers who are not familiar with denotational semantics can safely omit sections 3.1, 3.2, 3.4 and 3.5, and read only 3.3 and 3.6. In this case, two essential concepts that are required for 3.3 and 3.6 are that

- $\perp_D$ is the value of type D denoting "no information" (resulting perhaps from a non-terminating computation or a run-time error), and
- *continuous* functions are *monotonic* and *information-preserving* (i.e. applying the function to the least upper bound of a sequence of better-defined arguments is equivalent to applying the function to each argument individually and then taking the least upper bound of the results).

3.1 Background on Domains

If D is a set and $\sqsubseteq$ is a partial ordering on D, then for any subset $E \subseteq D$ there is at most one element $d \in D$ such that $\forall\, d' \in D,\ d \sqsubseteq d'$ if and only if $E \sqsubseteq d'$. If d exists, it is termed the *least upper bound* of E, written $\bigsqcup E$. The element $\bigsqcup \emptyset$ (if it exists) is denoted by $\perp$ and satisfies $\perp \sqsubseteq d,\ \forall\, d \in D$. D is said to be a *domain* if it has such a least element and if for every (possibly infinite) sequence of elements $d_1 \sqsubseteq d_2 \sqsubseteq \ldots$, their least upper bound $\bigsqcup\{d_1, d_2, \ldots\}$ also exists.

Given two domains, D_1 and D_2, a function $f{:}D_1 \rightarrow D_2$ is said to be *continuous* if, for every sequence $d_1 \sqsubseteq d_2 \sqsubseteq \ldots$ in D_1, $f(\bigsqcup\{d_1, d_2, \ldots\})$ is equal to $\bigsqcup\{f\, d_1, f\, d_2, \ldots\}$. In other words, continuous functions preserve least upper bounds. Functions are required to be continuous in order to guarantee the existence of a least fixed point. In particular, given a continuous function $f{:}D \rightarrow D$, the least fixed point of f exists and is given by

$$FIX(f) \equiv \bigsqcup\{\, f^n(\perp_D) \mid n \geq 0 \,\}$$

where $f^0(x) \equiv x$ and $f^i(x) \equiv f(f^{i-1}(x))$ for $i > 0$.

The simplest domain is a *flat domain* which has the property that if $d_1 \sqsubseteq d_2$ then either $d_1 = d_2$ or $d_1 = \perp_D$. It is possible to construct more complex domains from simpler domains. In particular, given domains D_1, ... D_n, four useful domains are the *product*, *sum*, *coalesced sum* and *continuous function* domains :

The *product* domain $D = D_1 \times D_2 \times \ldots \times D_n$ contains tuples of the form $<d_1, d_2, \ldots, d_n>$ where each $d_i \in D_i$ and $<d_1, d_2, \ldots, d_n> \sqsubseteq_D <e_1, e_2, \ldots, e_n>$ if $d_i \sqsubseteq_{D_i} e_i\ \forall\ 1 \leq i \leq n$. The bottom element is thus $<\perp_{D_1}, \ldots, \perp_{D_n}>$.

The *(separated) sum* domain $D = D_1 + D_2 + \ldots + D_n$ is defined as a union :

$$(\{1\} \times D_1) \cup (\{2\} \times D_2) \cup \ldots \cup (\{n\} \times D_n) \cup \{\perp_D\}$$

where $\perp_D \sqsubseteq_D d,\ \forall\ d \in D$, and $<i, d> \sqsubseteq_D <j, e>$ if the "tags" i and j are equal and $d \sqsubseteq_{D_i} e$.

The *coalesced sum* domain $D = D_1 \oplus \ldots \oplus D_n$ is also a union :

$$(\{1\} \times (D_1 - \{\perp_{D_1}\})) \cup (\{2\} \times (D_2 - \{\perp_{D_2}\})) \cup \ldots \cup (\{n\} \times (D_n - \{\perp_{D_n}\})) \cup \{\perp_D\}$$

where $\sqsubseteq_D$ is defined as for the separated sum. The coalesced sum is so called because the least element of each component space becomes identified with the least element of the sum space. This is in contrast to the separated sum where a new least element is introduced into the sum space.

The domain of *continuous functions* from a domain D_i to a domain D_j, $D = [D_i \rightarrow D_j]$, has $f \sqsubseteq_D g$ if $f(d) \sqsubseteq_{D_j} g(d),\ \forall\ d \in D_i$. The least element of D is thus $\lambda d.\perp_{D_j}$, the function which returns the least element of D_j for every element in D_i.

The equation defining a domain D may be recursive. An example is the domain of lists of integers, where NIL_{INT} is a distinguished constant :

$$LIST_{INT} = (INT \times LIST_{INT}) + \{NIL_{INT}\}$$

The solution of such recursive domain equations is discussed by Stoy [35].

3.2 Denotational Semantics of the PFL Kernel

Functional languages typically have a simple kernel into which expressions in the full syntax of the language are translated prior to evaluation. The semantics of the full language are thus derived from the semantics of the kernel. PFL follows this pattern and the syntax of its kernel is as follows :

```
expr    = id | const | expr expr |
          "(" expr "," ... "," expr ")" | λ id.expr |
          "fix" expr | expr "where" id "=" expr
```

In particular, the "fix" construct allows recursive functions to be defined. For example, the equation

```
fact x = if (x = 0) 1 (x*(fact(x-1)))
```

is syntactically transformed into the following non-recursive definition :

```
fact =
    fix λf.λx.if (x = 0) 1 (x*(f (x-1)))
```

Mutually recursive functions are handled by packaging them into a single non-recursive tuple similarly.

For our purposes, we assume that expressions are assigned values in the following semantic domain, D :

$$\begin{aligned} D = {} & W + \mathrm{Bool} + \mathrm{Atom} + [D \to D] + (D \times D) + \\ & (D \times D \times D) + \ldots + \mathrm{LIST}_I \end{aligned}$$

Here, Bool and Atom are flat domains representing booleans and other constants respectively, while LIST_I denotes an infinite number of domains such that each LIST_I consists of lists whose elements are drawn from domain I, e.g.

$$\begin{aligned} & \mathrm{LIST}_{\mathrm{Atom}} = \\ & \quad (\mathrm{Atom} \times \mathrm{LIST}_{\mathrm{Atom}}) + \{\mathrm{NIL}_{\mathrm{Atom}}\} \\ & \mathrm{LIST}_{\mathrm{Atom}\times\mathrm{Atom}} = \\ & \quad ((\mathrm{Atom} \times \mathrm{Atom}) \times \mathrm{LIST}_{\mathrm{Atom}\times\mathrm{Atom}}) + \{\mathrm{NIL}_{\mathrm{Atom}\times\mathrm{Atom}}\} \end{aligned}$$

In practice of course atoms are partitioned into separate domains in most functional languages (one exception is LISP) and lists are constrained to be homogeneous by the type checker. Type checking in PFL was discussed in [29] so we consider it no further here. We merely note that in the domain equation for D above, W contains a single element, "error", which is the value of any incorrectly typed expression.

Defining the denotational semantics of a language consists of identifying the *syntactic categories* of the language, and setting up *semantic functions* between these and the domain D (see [31, 35]). The three syntactic categories of our kernel are Con, Ide and Env $\equiv$ [Ide$\to$D], from which constructors, variables and environments are drawn, respectively. In particular, an environment ρ is a function that assigns to each variable in Ide a value in the semantic domain D. Two semantic functions are required to map expressions to values in D : the function K:[Con $\to$ D] maps each constructor to its predefined value, and the function E:[Exp $\to$ [Env $\to$ D]] maps any expression to a value, for a given environment. The definition of E is given below where, by convention, all expressions enclosed in square brackets, $[\![$ and $]\!]$, are in the syntax of the language whose semantics are being defined i.e. the kernel. All other expressions are in the defining notation, in this case some model of the λ calculus that supports λ abstraction (whence the Λ on the right hand side of the third equation) and function application (whence the $\bullet$ on the right hand side of the fourth equation). The notation ρ[d/id] denotes the environment obtained by extending ρ to map the variable id to the value d. As we would expect, the semantic function E mapping a kernel functional language to a model of the λ calculus is almost trivial. Specifically, variables are mapped to the value assigned to them by ρ, constants are mapped to the value assigned to them by K, λ abstractions are mapped to Λ abstractions, applications are mapped to applications, tuples are mapped to tuples, "fix" results in a fixpoint computation, and "where" clauses result in the extension of the environment with a new identifier :

$$\begin{aligned} & E[\![\,\mathrm{id}\,]\!]\rho && = \rho[\![\,\mathrm{id}\,]\!] \\ & E[\![\,\mathrm{const}\,]\!]\rho && = K[\![\,\mathrm{const}\,]\!] \\ & E[\![\,\lambda\ \mathrm{id.expr}\,]\!]\rho && = \Lambda\ \mathrm{d}.E[\![\,\mathrm{expr}\,]\!]\rho[\mathrm{d/id}] \\ & E[\![\,\mathrm{expr}_1\ \mathrm{expr}_2\,]\!]\rho && = (E[\![\,\mathrm{expr}_1\,]\!]\rho) \bullet (E[\![\,\mathrm{expr}_2\,]\!]\rho) \\ & E[\![\,(\mathrm{expr}_1,\ldots,\mathrm{expr}_n)\,]\!]\rho && = \langle E[\![\,\mathrm{expr}_1\,]\!]\rho,\ldots,E[\![\,\mathrm{expr}_n\,]\!]\rho\rangle \\ & E[\![\,\mathrm{fix\ expr}\,]\!]\rho && = \mathrm{FIX}\ (E[\![\,\mathrm{expr}\,]\!]\rho) \\ & E[\![\,\mathrm{expr}_1\ \mathrm{where\ id} = \mathrm{expr}_2\,]\!]\rho = \\ & \qquad E[\![\,\mathrm{expr}_1\,]\!]\rho[(E[\![\,\mathrm{expr}_2\,]\!]\rho)/\mathrm{id}] \end{aligned}$$

3.3 Adding Set Abstractions

We are now ready to add *set abstractions* to the kernel functional language. We need to extend first the syntax of the language to allow set expressions, then the semantic domain to include values for set expressions, and finally the semantic function E.

Set abstractions are similar to list abstractions in that they construct new sets from existing sets just as list abstractions construct new lists from existing lists. As well as set abstractions, we also introduce empty sets, singleton sets and set unions. The resulting language syntax is as follows :

```
expr        = id | const | expr expr |
              "(" expr "," ... "," expr ")" | λ id.expr |
              "fix" expr | expr "where" id "=" expr |
              "{}" | "{"expr"}" | expr "U" expr |
              "{" expr "|" qualifiers "}"
qualifiers  = qualifier | qualifier "&" qualifiers
qualifier   = generator | filter
generator   = id "∈" expr
filter      = expr
```

We also allow the notation { $s_1, \ldots, s_n$ } in lieu of {s_1} U ... U {s_n}. Two examples of set expressions are

```
{(John,Jane),(Jack,Jane)}
{n | n ∈ nat & (n mod 2) = 0}
```

where the second expression denotes the set of even numbers in nat. In 3.5 below we extend this syntax to allow arbitrary patterns on the left-hand side of generator expressions, so that the following set abstractions respectively return (i) all the tuples in the set parents, (ii) all the tuples in the set parents which have Bill as their first component, and (iii) the singleton set {(Bill,Mary)} if this tuple is in the set parents, and the empty set otherwise:

```
{(x,z) | (x,z) ∈ parents}
{(Bill,z) | (Bill,z) ∈ parents}
{(Bill,Mary) | (Bill,Mary) ∈ parents}
```

The semantic domain D of 3.2 is extended to include an infinite number of domains SET_I such that each SET_I consists of sets whose elements are drawn from domain I:

$$\begin{aligned} D = {} & W + \mathrm{Bool} + \mathrm{Atom} + [D \to D] + (D \times D) + \\ & (D \times D \times D) + \ldots + \mathrm{LIST}_I + \mathrm{SET}_I \end{aligned}$$

Three main orderings have been proposed for set domains in the literature : the Hoare, Smyth and Plotkin orderings (the last of these is also known as the Egli-Milner ordering), and a comparison of them is given in [34]. Of these, the Hoare and Plotkin orderings are both candidates for our purposes since they consider larger sets to be better defined than smaller sets. The Hoare ordering permits sets to contain an infinite number of non-$\perp_I$ elements whereas under the Plotkin ordering sets are either finite or contain $\perp_I$. We have chosen to use the Plotkin construction rather than the Hoare one since, as we will see below, this allows us to map from sets to lists and thereby to fully integrate sets into PFL. This is not possible with the Hoare ordering (which was adopted by Silbermann and Jayaraman in their integration of functional and logic programming [32]) since such a mapping would not be continuous. More specifically, the Plotkin ordering, $\sqsubseteq_{Plot}$, on two sets S and T whose elements are drawn from a flat domain I is defined as follows :

$$S \sqsubseteq_{Plot} T \text{ iff}$$
$$\forall s \in S. \exists t \in T. s \sqsubseteq_I t \text{ and } \forall t \in T. \exists s \in S. s \sqsubseteq_I t$$

(The definition for non-flat I is more intricate and can be found in [31]). Thus, the least element is $\{\perp_I\}$ and the empty set is not part of the domain. This is because the Plotkin powerdomain is traditionally used to model non-determinism, where all computations have *some* result even if this result is non-termination. In our case we wish to model set-valued functions and so the empty set *is* meaningful. The structure of SET_I is thus the coalesced sum (see 3.1) of the Plotkin powerdomain with a singleton domain representing the empty set of type I.

For example, the left-hand part of Figure 1 below shows the structure of SET_{Bool}. Computation over this domain commences at the least element $\{\perp_{Bool}\}$ and proceeds either to the empty set or to a non-empty set. In the latter case the set may contain the element $\perp_{Bool}$, representing the possibility that more elements will be added to the set or that a non-terminating computation will occur. When no more elements can be added to a set, computation terminates with a set not containing $\perp_{Bool}$.

To stay within a purely functional formalism (and thereby continue to benefit from the advantages of functional programming we discussed in the introduction) there are two key requirements for operations over sets : determinism and continuity. To retain determinism we assume a total ordering, <, on all elements of Atom and Bool except $\perp_{Atom}$ and $\perp_{Bool}$, which in turn induces a total ordering on all finite elements of D (except the functions in [D→D] of course). We then use an operator

$$set\_to\_list_I : SET_I \rightarrow LIST_I$$

which maps elements of SET_I to elements of $LIST_I$ (we often write set_to_list without its subscript since this can be inferred from context). It does this by taking

- any set containing a $\perp$ to $\perp_{LIST_I}$, and
- any other set to the (unique) list whose components are precisely those of the set and appear in the order of < without duplicates.

For example, Figure 1 below shows the SET_{Bool} and $LIST_{Bool}$ domains and the $set\_to\_list_{Bool}$ mapping between them (which assumes that Any_{Bool} < False < True). We note that set_to_list is continuous, as required.

3.4 Denotational Semantics of Set Expressions

We now extend the semantic function E of 3.2 to map set expressions into sets. Before doing so we require three standard (and continuous) operators from powerdomain theory [31]. The *singleton* operator constructs singleton sets in SET_I from elements in I:

$$\{\_\} : I \rightarrow SET_I$$

The *union* operator constructs new sets from pairs of sets:

$$\_\cup\_ : (SET_I \times SET_I) \rightarrow SET_I$$

Finally the + operator distributes functions of type $I \rightarrow SET_J$ over sets of type SET_I and takes the union:

$$\_+ \; : (I \rightarrow SET_J) \rightarrow (SET_I \rightarrow SET_J)$$

In particular, for any function $F: I \rightarrow SET_J$, the function $F+: SET_I \rightarrow SET_J$ is defined as follows:

$$F+ \; S = \cup \{ \textbf{if } d = \perp_I \textbf{ then } \{\perp_J\} \textbf{ else } (F\ d) \mid d \in S\}$$

The extended semantic function E assigns meanings to set expressions as follows, where we have explicitly subscripted set expressions with the type of their components where necessary:

$$E[\![\{\}_I]\!]\rho = \emptyset_I$$
$$E[\![\{expr\}_I]\!]\rho = \{ E[\![expr]\!]\rho \}_I$$
$$E[\![expr_1 \cup expr_2]\!]\rho = (E[\![expr_1]\!]\rho) \cup (E[\![expr_2]\!]\rho)$$
$$E[\![\{expr \mid filter\text{-}expr \ \&\ rest\}_I]\!]\rho = \textbf{if } E[\![filter\text{-}expr]\!]\rho \textbf{ then } E[\![\{expr \mid rest\}]\!]\rho \textbf{ else } \emptyset_I$$
$$E[\![\{expr \mid id \in set\text{-}expr \ \&\ rest\}]\!]\rho = (\Lambda\, d.E[\![\{expr \mid rest\}]\!]\rho[d/id]) + \bullet (E[\![set\text{-}expr]\!]\rho)$$

Thus, singleton expressions map to singleton sets, union expressions map to unions of sets, filters map to conditionals, and generators id ∈ set-expr map to functions which generate bindings for the variable id by being distributed over set-expr.

We conclude this section by noting that the ?= operator of section 2.2 can be extended to match set terms in a number of ways. Three possibilities are $?=_H$, which simulates the Hoare ordering on sets (i.e. every element in the left-hand operand of $?=_H$ must match some element of the right-hand operand) :

$$S1 \; ?=_H \; S2 \;=\; \{p \mid p \in S1 \;\&\; v \in S2 \;\&\; p ?=_H v\} = S1$$

$?=_S$, which simulates the Smyth ordering (i.e. every element of the right-hand operand must be matched by some element of the left-hand operand) :

$$S1 \; ?=_S \; S2 \;=\; \{v \mid v \in S2 \;\&\; p \in S1 \;\&\; p ?=_S v\} = S2$$

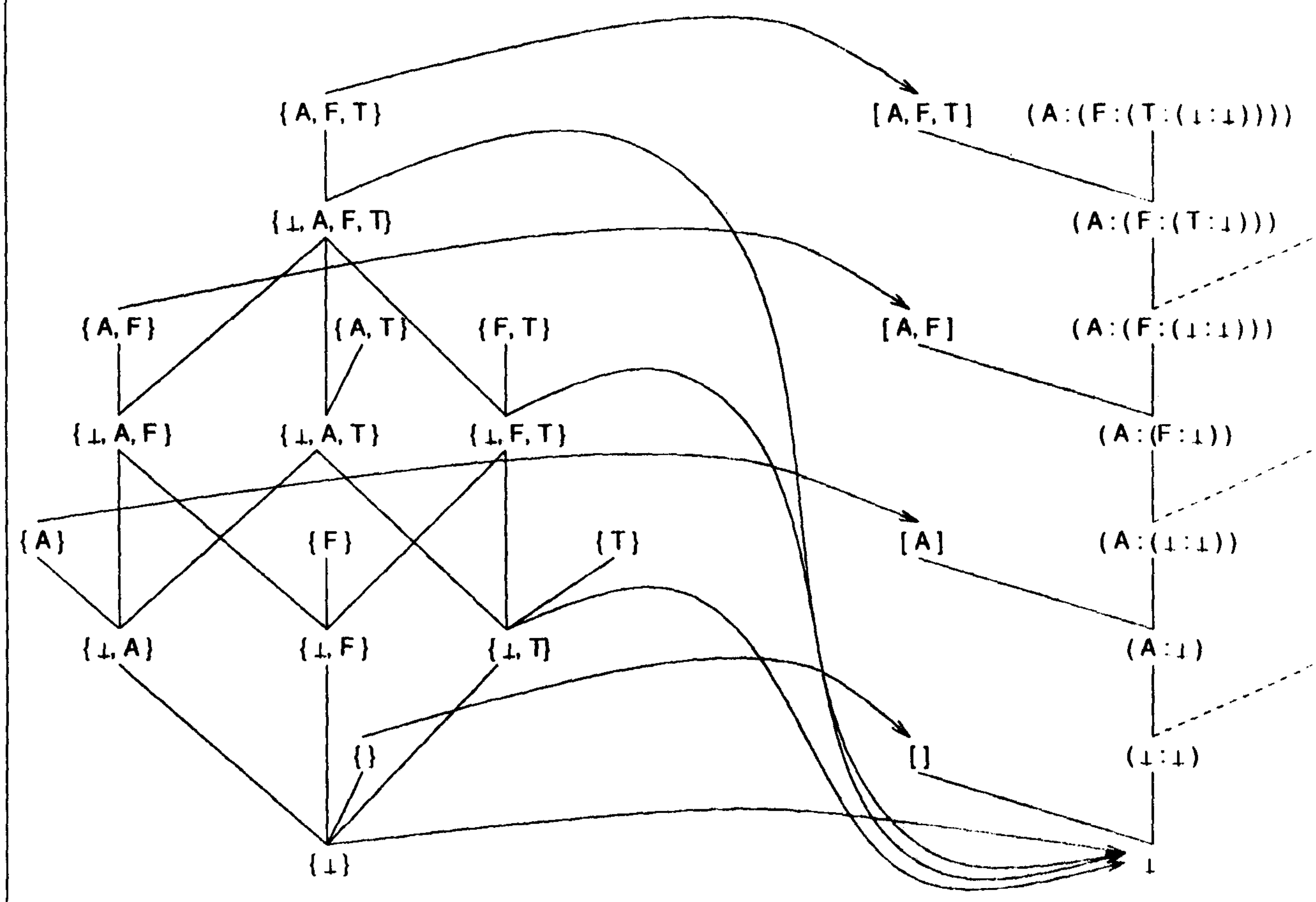

Figure 1 : Mapping from Set_{Bool} to $List_{Bool}$, where A ≡ Any_{Bool}, T ≡ True and F ≡ False.

and $?=_P$, which simulates the Plotkin ordering (i.e. every element of the left-hand operand must match some element of the right-hand operand, and every element of the right-hand operand must be matched by some element of the left-hand operand) :

```
S1 ?=P S2  =
     ({p | p∈S1 & v∈S2 & p?=Pv} = S1)
     and
     ({v | v∈S2 & p∈S1 & p?=Pv} = S2)
```

3.5 Extending the Kernel to the Full Language

In common with most functional programming languages, PFL allows *patterns* (i.e. expressions in constants and variables) in several places where the kernel syntax allows only variables. These are : λ abstractions, left hand sides of equations, and left hand sides of generators in list and set abstractions. Peyton Jones [27] discusses how such features can be translated into a kernel language, although not for patterns in set abstractions. So for completeness we give the semantics of this feature. To do so we need to add one more equation to the definition of *E* in order to take care of generators with patterns i.e. generators of the form c pat_1 ... pat_n ∈ set-expr, where c is a constructor with $n \geq 0$ arguments which are themselves variables or further patterns. The effect of such generators is to iterate over only those elements of set-expr that match the pattern c pat_1 ... pat_n. In our semantic equation for this feature below we assume the availability of pattern-matching λ abstractions [27] i.e. expressions of the form λ pat.expr which when applied to an argument, arg, substitute in expr the variables of pat by constants from arg if pat matches arg and otherwise return the constant error ∈ W (of section 3.2) :

E❙ {expr | c pat_1...pat_n ∈ set-expr & rest}$_1$ ❙ρ =
  (Λ d.**if** (E❙ λ c pat_1...pat_n.{expr|rest} ❙ρ) • d ≠ error
    then (E❙ λ c pat_1...pat_n.{expr|rest} ❙ρ) • d
    else $\varnothing_1$)+ • (E❙ set-expr ❙ρ)

3.6 Bulk Data

PFL's bulk data is stored in a class of functions called *selectors*. In [29, 33] we defined selectors as list-valued functions defined by an equation of the form

```
f p = [v | v <- relation; p ?= v]
```

where the relation was a list without duplicates and where updates to the selector could change the order of its elements. Having now incorporated sets into PFL we can more properly formalise selectors as set-valued functions. In particular, a selector is now declared by a statement of the form

"selector" name "::" type ";"

where the type is required to be of the form $\iota \rightarrow \{\iota\}$ for some first-order monomorphic type ι. For example, the following statement declares a selector that can be used to store details of books - consisting of their ISBNs, titles, authors' names and year of publication :

```
selector books ::
      (Num,Str,[Str],Num) →
      {(Num,Str,[Str],Num)};
```

A newly declared selector, f, may be assumed to be defined by an equation of the form

```
f p = {v | v ∈ relation & p ?= v}
      where relation = {}
```

Updates to a selector result in the relation expanding or contracting. In particular, the command

"include" selector value ";"

adds a new value to the relation, while the command

"exclude" selector pattern ";"

removes from the relation all the values that match the pattern. For example, the inclusions

```
include books (071678158,
      "Principles of Database and
       Knowledge-Base Systems",
      ["J.D.Ullman"],
      1988);
include books (052126896,
      "Introduction to Combinators and
       Lambda Calculus",
      ["J.R.Hindley","J.P.Seldin"],
      1986);
include books (020508974,
      "Denotational Semantics",
      ["D.A.Schmidt"],
      1986);
```

result in the following definition for the books selector :

```
books p = {v | v ∈ relation & p ?= v}
      where relation =
      {(071678158,"Principles...",...),
      (052126896,"Introduction...",...),
      (020508974,"Denotational...",...)}
```

and subsequently the exclusion

```
exclude books (Any,Any,Any,1986)
```

removes the Hindley & Seldin and Schmidt books.

The application of a selector to an argument returns the set of the values in the relation that match the argument. For example `books Any` returns the entire relation while `books (Any,Any,Any,1986)` returns the details of books published in 1986.

4. Implementation

According to the definition of the fixpoint operator FIX in 3.1, functional languages can evaluate functions by commencing with least elements everywhere and continuing until no more information is inferred. For example, the factorial function, fact, of 3.2 is the least fixed point of the following higher-order function

$$H \;=\; \lambda f.\lambda x.\text{if } (x=0)\ 1\ (x * (f\,(x-1)))$$

By the definition of FIX the successive approximations to H are as follows :

H^0 maps all numbers to $\perp_{Num}$,

H^1 maps all numbers to $\perp_{Num}$ except 0 which is mapped to 1,

H^2 maps all numbers to $\perp_{Num}$ except 0 and 1 which are both mapped to 1,

H^3 maps all numbers to $\perp_{Num}$ except 0 and 1 which are both mapped to 1, and 2 which is mapped to 2,

and so on, obtaining the factorial of a (positive) number i on the $i{+}1^{th}$ approximation.

However, the expression $Y \equiv \lambda h.(\lambda x.h(x\ x))\ (\lambda x.h(x\ x))$ has the property that for any expression H, (Y H) = H (Y H) i.e. (Y H) is a fixed point of H. In fact it can be shown [35] that (Y H) is the *least* fixed point of any continuous function H i.e. (Y H) denotes FIX(H). Thus, functional languages typically evaluate functions top-down using Y rather than bottom-up using FIX. For example, to evaluate the factorial of 2 we start off with the expression (Y H) 2, and this is simplified through the following reductions :

$$\begin{aligned}
&(Y\,H)\,2\\
&\rightarrow H\,(Y\,H)\,2\\
&\rightarrow (\lambda x.\text{if } (x=0)\ 1\ (x * ((Y\,H)(x-1))))\ 2\\
&\rightarrow 2 * ((Y\,H)(2-1))\\
&\rightarrow 2 * ((Y\,H)\,1)\\
&\rightarrow 2 * (H\,(Y\,H)\,1)\\
&\rightarrow 2 * ((\lambda x.\text{if } (x=0)\ 1\ (x * ((Y\,H)(x-1))))\ 1)\\
&\rightarrow 2 * (1 * ((Y\,H)\,(1-1)))\\
&\rightarrow 2 * (1 * ((Y\,H)\,0))\\
&\rightarrow 2 * (1 * (H\,(Y\,H)\,0))\\
&\rightarrow 2 * (1 * (((\lambda x.\text{if } (x=0)\ 1\ (x * ((Y\,H)(x-1))))\ 0)\\
&\rightarrow 2 * (1 * 1)\\
&\rightarrow 2
\end{aligned}$$

Operationally then, Y has the affect of producing a new copy of (Y H) (the "meaning" of the factorial function) upon each recursive call. Notice that we are chosing to simplify the left-most, outermost redex at each step above - this computation rule is known as *normal-order reduction* and it corresponds to the *lazy evaluation* of function arguments. It is necessary to adopt normal-order reduction in order to ensure that (Y H) terminates.

Evaluation in the presence of sets can proceed in a similar fashion. Sets are implemented as binary trees with the $\cup$ operator at the inner nodes and singletons at the leaves. The operator + applies a function F to a set by distributing F to all the leaves. The application of F to singletons can occur asynchronously e.g. in parallel. However, the operator + violates lazy evaluation since it evaluates the argument to F before applying F (see the definition of F+ in 3.4). Thus, although all the elements of the set are eventually discovered, the top-down evaluation of a set-valued function may fail to terminate (c.f. the difficulties in detecting termination when logic rules are evaluated top-down). At present we therefore use a bottom-up evaluation strategy for recursive (and mutually-recursive) set-valued functions. This strategy builds up sets "naively" for "interesting" arguments of the function. Optimisation to utilise semi-naive evaluation techniques is an ongoing area of work.

5. Expressiveness

Selectors are the analogue of the EDB predicates of a Datalog database, and from now on we term them *extensional* selectors. In 5.1 below we introduce the concept of an *intentional* selector and show how any Datalog IDB predicate - including those whose rules include negation and function symbols - can be translated into a PFL intentional selector. In 5.2 we discuss the optimisation of equations defining intentional selectors. In 5.3 we show how PFL with sets supersedes Datalog by supporting aggregation and counting operations. Lastly, in 5.4 we show how integrity constraints over selectors can be defined and efficiently enforced.

5.1 Intentional Selectors

We define an *intentional selector* to be a function f defined by an equation of the form

$$f\ p = s_1 \cup \ldots \cup s_n$$

where each s_i is either an application of a selector to the pattern p, or a set abstraction $\{p \mid q_1 \ \& \ \ldots \ \& \ q_n\}$ all of whose generators iterate over selectors. For example, the following function is an intentional selector :

```
anc (x,z) =
     parent (x,z) U
     {(x,z) | (x,y1) ∈ parent (x,Any) &
              (y2,z) ∈ anc (Any,z) &
              y1 = y2}
```

In our translation of Datalog rules into PFL, we first consider the case of IDB predicates defined by a single rule of the following form, where each q_i is either an EDB or IDB predicate, and each r_j is a built-in predicate :

$$p(\bar{x}) \leftarrow q_1(\bar{y}_1), \ldots, q_n(\bar{y}_n), r_1(\bar{y}_{n+1}), \ldots, r_m(\bar{y}_{n+m})$$

Without loss of generality we assume that the variables of $\bar{x}$ are distinct. We also make the usual assumption of *range-restriction* [36] for the r_j i.e. any variable appearing as an argument to an r_j is instantiated by some q_i. We translate such a rule into a PFL equation of the form

$$p\,\bar{x} = \{\bar{x} \mid \ a_1 \ \& \ldots \& \ a_n \ \& \ b_1 \ \& \ldots \& \ b_m \ \& \ c_1 \ \& \ldots \& \ c_p\}$$

where :

(i) each literal $q_i(y_{i\,1}, y_{i\,2}, \ldots)$ in the rule results in a generator a_i of the form $(v_{i\,1}, v_{i\,2}, \ldots) \in q_i(w_{i\,1}, w_{i\,2}, \ldots)$ such that
  - a constant or function symbol in a $y_{i\,j}$ maps to the same constructor in $v_{i\,j}$ and $w_{i\,j}$,
  - a universally quantified variable in a $y_{i\,j}$ maps to the same variable in $v_{i\,j}$ and $w_{i\,j}$, and
  - an existentially quantified variable in a $y_{i\,j}$ maps to a new variable in $v_{i\,j}$ and the constant Any in $w_{i\,j}$.

(ii) each literal $r_i(\bar{y}_{n+i})$ in the rule results in a semantically identical filter b_i in the equation; and

(iii) each pair of new variables $z_{i\,j}$ and $z_{i'\,j'}$ arising from the same existentially quantified variable in (i) result in a filter c_k of the form $z_{i\,j} = z_{i'\,j'}$.

We note that the equation resulting from this translation satisfies the criteria for an intentional selector. For example, the following Datalog rule finds proper siblings:

```
sibling(X,Y) <-
     parent(Z,X), parent(Z,Y), X ≠ Y
```

and the equivalent PFL equation obtained by our translation scheme is

```
sibling(x,y) =
     {(x,y) | (z1,x) ∈ parent(Any,x) &
              (z2,y) ∈ parent(Any,y) &
              x != y  &  z1 = z2}
```

We note that the repeated occurrences of the variables x and y in this equation are not ambiguous since, by the semantics of set abstractions, variable bindings are inherited initially from the left hand side of the equation or from the first generator with the variable in its head, and are then overidden by subsequent occurrences in the heads of generators. This process of inheriting bindings from one qualifier to the next is, of course, the precise analogue of *sideways information passing* [36] in the evaluation of logic rules.

The above translation scheme is clearly inefficient : for example, rules containing existentially quantified variables translate into equations which take cartesian products of selectors and then perform selections. More sophisticated translation schemes which overcome these problems are possible (e.g. the one given in [29] for the previous version of PFL). However, we have used this particular scheme here because it simplifies our discussion of selector optimisation in 5.2 below.

We can extend the translation scheme to IDB predicates that are defined by more than one rule. Without loss of generality we assume that these rules are *rectified* [36] i.e. that they all have the same head. Each rule is first separately translated according to the above scheme, giving a number of equations with the same left hand side. These equations are then combined into a single equation by forming a union from their right hand sides. For example, the standard rules for the ancestor predicate

```
anc(X,Z) ← parent(X,Z)
anc(X,Z) ← anc(X,Y), anc(Y,Z)
```

translate into the PFL equation

```
anc (x,z) =
     parent (x,z) U
     {(x,z) | (x,y1) ∈ anc(x,Any) &
              (y2,z) ∈ anc(Any,z) &
              y1 = y2}
```

Similarly, the rules

```
nat(X) ← is_number(X)
nat(X) ← nat(Y), X = succ(Y)
```

translate into the equation

```
nat x =
     {x | x ∈ is_number x} U
     {x | y ∈ nat Any & x ∈ {Succ y}}
```

assuming that is_number is a selector of type Nat→{Nat} and Succ a constructor of type Nat→Nat.

We now extend our translation scheme to cover the case of negative literals. We first make the usual assumption that every variable appearing in a negative literal also appears in some positive literal which precedes it in the rule. We then translate a negative literal, $\neg p(\bar{z})$, into a filter $(p\ \bar{z}) = \{\}$. For example, the following Datalog rule assumes that people are male if they are not female

```
male(X) ← person(X) & ¬female(X)
```

and the equivalent PFL equation is

```
male x =
     {x|x ∈ person x & (female x)={}}
```

The translation of Datalog rules which are not stratified with respect to recursion through negation results in functions which yield only the least element of the set domain. For example, this occurs if we define males as persons who are not female and vice versa :

```
male x =
     {x|x ∈ person x & (female x)={}}
female x =
     {x|x ∈ person x & (male x)={}}
```

A similar source of no information being inferrable is recursion through the set_to_list function of 3.3 (recall that set_to_list requires its argument to be fully evaluated before it can be applied).

5.2 Transformation of Intentional Selectors

It is possible to apply several syntactic transformations to the set abstractions defining intentional selectors. In particular, we use the following transformations :

Tr1: Reordering the qualifiers so that each filter appears as early as possible and each generator as late as possible, subject to the constraint that each qualifier must follow all of the generators which provide instantiations for its free variables (this is equivalent to undertaking selections as early as possible in relational queries).

Tr2: Switching pairs of generators over the same selector, $p_a \in s\ \bar{a}$ and $p_b \in s\ \bar{b}$, whenever $\bar{a}$?= $\bar{b}$ (subject to the same constraint as for Tr1).

Tr3: Replacing equality by pattern-matching (this is equivalent to pushing selections through joins) i.e. given a sequence of qualifiers

$$... \ \&\ q_{n-1}\ \&$$
$$(..,x_{i-1},x_i,x_{i+1},...) \in s\ (...,y_{i-1},Any,y_{i+1},...)\ \&$$
$$q_{n+1}\ \&\ ...\ \&\ q_{n+m-1}\ \&\ E = x_i\ \&\ q_{n+m+1}\ \&\ ...$$

where E is an expression all of whose variables are bound by qualifiers up to q_{n-1}, we can remove the qualifier $E = x_i$ and replace Any by E in q_n :

$$... \ \&\ q_{n-1}\ \&$$
$$(..,x_{i-1},x_i,x_{i+1},...) \in s\ (...,y_{i-1},E,y_{i+1},...)\ \&$$
$$q_{n+1}\ \&\ ...\ \&\ q_{n+m-1}\ \&\ q_{n+m+1}\ \&\ ...$$

For example, applying the anc function of 5.1 above to an argument (Any,B), where B is a constant other than Any, results in the equation :

```
anc (Any,B) =
     parent (Any,B) U
     {(x,z) | (x,y1) ∈ anc (Any,Any) &
              (y2,z) ∈ anc (Any,B) &
              y1 = y2}
```

By applying Tr2 we obtain

```
anc (Any,B) =
     parent (Any,B) U
     {(x,z) | (y2,z) ∈ anc (Any,B) &
              (x,y1) ∈ anc (Any,Any) &
              y1 = y2}
```

and by then applying Tr3 we obtain

```
anc (Any,B) =
     parent (Any,B) U
     {(x,z) | (y2,z) ∈ anc (Any,B) &
              (x,y1) ∈ anc (Any,y2)}
```

The process we have followed resembles the magic sets [6] method of rewriting Datalog rules, which generates a different set of rules for a predicate depending upon the binding pattern. We are currently investigating adapting the full magic sets method for the bottom-up evaluation of intentional selectors. A further optimisation would be to store evaluated intentional selectors in the database until an update to a *dependent function* (see 5.4 below) invalidates them.

5.3 Beyond Datalog

As we would expect, PFL is more expressive than Datalog since set-manipulation functions such as nesting, counting, and unnesting can be defined. We give three examples below : "nest" nests the anc relation, yielding a set of pairs (x,ancs) such that ancs is the set of all the ancestors of person x; "generation" returns a set of pairs (x,g) such that g is the generation of person x (which is calculated by counting the number of female ancestors of x); and "unnest" yields a set of pairs (x,a) such that a is an ancestor of x :

```
nest =
     {(x,{a|(a,x) ∈ anc (Any,x)}) |
      (y,x) ∈ parent (Any,Any)}
generation =
     {(x,length$ {y|y ∈ z &
                         female y != {}}) |
      (x,z) ∈ nest}
unnest =
     {(x,a)|(x,ancs) ∈ nest & a ∈ ancs}
```

The second of these functions uses an infix operator $ which allows a function expecting a list as an argument to be applied to a set; $ is defined as

```
f$ s= f (set_to_list s)
```

5.4 Integrity Constraints over Selectors

We define an *integrity constraint* to be an intentional selector, f, such that the database state is incorrect whenever the expression (f Any) denotes anything other than the empty set. For example, we may define a constraint ic1 which states that no-one is both male and female and a constraint ic2 which states that no-one is their own ancestor :

```
ic1 x =
     {y1 |    y1 ∈ male x &
              y2 ∈ female x &
              y1 = y2}
ic2 x =
     {x |     (x,y) ∈ anc (x,Any) &
              x = y}
```

Since the filters of set abstractions in an intentional selector can contain arbitrary functions, we can also enforce cardinality constraints. For example, ic3 ensures that anyone who is recorded as having parents (the first generator) has precisely two parents (the second generator) :

```
ic3 x =
     {x |(y,x) ∈ parent (Any,x) &
          length$ (parent (Any,x))!=2}
```

It is clearly desirable to use the updates to the database to restrict the amount of computation required to validate constraints. A number of constraint optimisation techniques have been developed for logic languages (see [15] for a review of them) any of which could be adapted for our integrity constraints. Here we show how one such method - that of Lloyd and Topor [21] - can be used. Our account roughly follows that of [30] for the handling of integrity constraints in the previous version of PFL, modified to utilise set-valued rather than list-valued constraints.

We say that a function f1 *directly depends upon* a function f2 if f2 occurs in an equation that defines f1; we use the term *depends upon* for the transitive closure of the *directly depends upon* relationship. This "call graph" is already automatically maintained by PFL as equations are inserted and deleted since we need it to infer the types of functions. We can now put it to further use for the enforcement of integrity constraints.

We recall from 5.1 that an intentional selector (and hence an integrity constraint) is a function of the form :

$$f\ p = \{c_1 \mid q_{1\,1}\ \&\ \dots\} \cup \dots \cup \{c_m \mid q_{m\,1}\ \&\ \dots\}$$

where, without loss of generality, we have converted summands of the form s p in the syntax of 5.1 to set abstractions of the form {p | p ∈ s p}. We distinguish between three different forms of qualifier occurring in such an intentional selector :

Type 1 : $q_{i\,j}$ is of the form p1 ∈ s p2,

Type 2 : $q_{i\,j}$ is of the form s p = {}, and

Type 3 : any other form of qualifier,

where s is a selector and p, p1 and p2 are patterns. Note that the three types of qualifier correspond to positive, negative and built-in predicates of Datalog rules respectively. Inclusion or exclusion of tuples in extensional selectors trigger implicit inclusions and exclusions from intentional selectors. In particular, the (explicit or implicit) inclusion of a value $t1 = (u_1,\dots,u_r)$ into one selector, s1, may affect another selector, s2, in one of three ways :

(i) *Inclusion of values into s2* : If s2 directly depends upon s1 via a Type 1 qualifier i.e.

$$s2\ p = \dots \cup \{c \mid \dots\ \&\ (v_1,\dots,v_r) \in s1\ (w_1,\dots,w_r)\ \&\ \dots\} \cup \dots$$

let t be the value obtained from c by replacing v_j by u_j, for all $1 \le j \le r$, and any other free variables of c by Any. Then for any value t2 implicitly included into s2 as a result of the insertion of t1 into s1, we have t ?= t2 holds.

(ii) *Exclusion of values from s2* : If s2 directly depends upon s1 via a Type 2 qualifier i.e.

$s2\ p = \ldots \cup \{e \mid \ldots \ \&\ s1\ (v_1,\ldots,v_r) = \{\ \}\ \&\ \ldots\} \cup \ldots$

again, let t be the value obtained from e by replacing v_j by u_j, for all $1 \le j \le r$, and any other free variables of e by Any. Then for any tuple t2 implicitly excluded from s2 as a result of the insertion of t1 into s1, we have t ?= t2.

(iii) *Both inclusion into and exclusion from s2* : If s2 depends upon s1 via a Type 3 qualifier then arbitrary values may implicitly have been included and excluded. Let t be the value obtained from e by replacing any free variables by Any. Then for any tuple t2 implicitly included or excluded from s2, we have t ?= t2.

The exclusion of a value from one selector may similarly affect another : in this case the roles of Type 1 and Type 2 qualifiers are interchanged. Thus, in our description of the constraint enforcement algorithm below, we assume the availability of two functions : "includes(s,v,u)" takes a selector s, a value v and a flag u indicating whether v represents a possible inclusion or a possible exclusion from s, and returns a set of pairs (s′,v′) such that s′ is a selector either directly depending upon s via a Type 1 or Type 2 qualifier or indirectly via a Type 3 qualifier, and the v′ match all the inclusions into the s′; similarly, "excludes(s,v,u)" returns a set of pairs (s′,v′) such that the v′ match all exclusions from the s′.

To optimise the enforcement of constraints, the system automatically associates with each selector, s, affected by an update two sets, s_i and s_e, which respectively contain tuples matching all tuples included into (or excluded from) s. These sets are built recursively using the function "update" given below. In particular, after the inclusion (or exclusion) of a value into an extensional selector s, update(s,v,"i") (or update(s,v,"e")) is called to maintain these sets :

```
update(s,v,i_or_e)
{
if v ∈ s_i_or_e
    return;
s_i_or_e = s_i_or_e U {v};
for ((s′,v′) ∈ includes(s,v,i_or_e))
    update(s′,v′,"i");
for ((s′,v′) ∈ excludes(s,v,i_or_e))
    update(s′,v′,"e");
}
```

The database is validated with respect to the constraints when a commit point is reached. Validity is ascertained by evaluating the expression (ic t) for each constraint ic and each value t in ic_i. In general, a set ic_i may contain redundant tuples : whenever there are tuples t1, t2 $\in ic_i$ such that t1 ?= t2, then t2 may safely be removed from the set since ic t2 $\subseteq$ ic t1. Thus constraints need only be validated with respect to these reduced sets of tuples.

6. Conclusions

We have described how the functional database language PFL can be extended with sets as first class objects. Our support of the value Any and the ?= operation then allows any $\text{Datalog}^{fun+neg}$ predicate to be expressed as a PFL function. We thus combine the respective advantages of functional and logic database languages within one semantic and operational framework. Our work can also be considered as contributing to the formalisation of database concepts using powerdomain theory, as exemplified by Buneman *et al* [8].

More specifically, in common with functional database languages such as [3, 4, 5, 7, 16, 18, 23, 28] we support deterministic computations over large volumes of data. We also support the storage of all types and functions in the database, a feature found only in [28]. In common with logic database languages [12, 13, 25] we also support search-based computations over large volumes of data. Several functional languages provide relational processing by incorporating records [2, 9, 23, 24, 26] but it is not clear how full deductive capabilities can be achieved in these languages. FAD [4] does add sets to a functional computation model, but relies upon the ability to "call out" to an external, computationally complete, language in order to define arbitrary functions. Several logic-based languages also incorporate sets [20]. In particular, COL [1, 17] integrates both functions and sets into a logic framework and thus has similar expressiveness to our language. However, it too achieves this by assuming the ability to call out to an external language to define arbitrary functions. In contrast we use one language and one database to store all information.

We have indicated how optimisation techniques developed for both functional and logic languages can be transferred to PFL, for example for recursive query processing and for integrity constraint enforcement. These are areas of on-going research. We are also investigating suitable bulk data structures to efficiently support the ?= operation. Finally, PFL is currently being used to analyse road traffic accident data, which requires both search-based computation, e.g. to find the nearest site (junction, roundabout etc.) to a given accident location, and deterministic computation, e.g. to group accidents by site and to produce accident statistics.

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